



TEXAS EDUCATION AGENCY

TEXAS EDUCATOR CERTIFICATION

TEXES

TexMaT

TASC/TASC-ASL

# Texas Examinations of Educator Standards™ (TEXES™) Program

## Preparation Manual

### Physics/Mathematics 7–12 (243)



---

## Table of Contents

About The Test.....	4
The Domains.....	5
The Standards.....	6
Domains and Competencies .....	8
Domain I — Number Concepts.....	8
Domain II — Patterns and Algebra .....	10
Domain III — Geometry and Measurement.....	14
Domain IV — Probability and Statistics .....	16
Domain V — Mathematical Processes and Perspectives.....	19
Domain VI — Mathematical Learning, Instruction and Assessment.....	20
Domain VII — Scientific Inquiry and Processes.....	21
Domain VIII — Physics .....	24
Domain IX — Science Learning, Instruction and Assessment.....	27
Approaches to Answering Multiple-Choice Questions .....	30
How to Approach Unfamiliar Question Formats.....	31
Question Formats .....	32
Single Questions.....	32
Clustered Questions .....	34
Multiple-Choice Practice Questions .....	41
Definitions and Formulas for Mathematics 7–12 .....	42
Definitions and Physical Constants for Physics 7–12.....	43
Answer Key and Rationales .....	63

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Study Plan Sheet .....	84
Preparation Resources .....	85

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## About The Test

Test Name	Physics/Mathematics 7–12
Test Code	243
Time	5 hours
Number of Questions	120 multiple-choice questions
Format	Computer-administered test (CAT)

The TExES Physics/Mathematics 7–12 (243) test is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 120 multiple-choice questions are based on the Physics/Mathematics 7–12 test framework. Questions on this test range from grades 7–12. The test may contain questions that do not count toward the score.

The number of scored questions will not vary; however, the number of questions that are not scored may vary in the actual test. Your final scaled score will be based only on scored questions.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## The Domains

Domain	Domain Title	Approx. Percentage of Test	Standards Assessed
I.	Number Concepts	7%	Mathematics I
II.	Patterns and Algebra	16%	Mathematics II
III.	Geometry and Measurement	10%	Mathematics III
IV.	Probability and Statistics	7%	Mathematics IV
V.	Mathematical Processes and Perspectives	5%	Mathematics V–VI
VI.	Mathematical Learning, Instruction and Assessment	5%	Mathematics VII–VIII
VII.	Scientific Inquiry and Processes	7%	Physical Science I–III, VI–VII, XI
VIII.	Physics	39%	Physical Science VIII
IX.	Science Learning, Instruction and Assessment	4%	Physical Science IV–V

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## The Standards

### **Mathematics Standard I**

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

### **Mathematics Standard II**

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

### **Mathematics Standard III**

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

### **Mathematics Standard IV**

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

### **Mathematics Standard V**

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

### **Mathematics Standard VI**

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

### **Mathematics Standard VII**

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts, knows typical errors students make and uses this knowledge to plan, organize and implement instruction; to meet curriculum goals and to teach all students to understand and use mathematics.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

**Mathematics Standard VIII**

Mathematical Assessment: The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

**Physical Science Standard I**

The science teacher manages classroom, field and laboratory activities to ensure the safety of all students and the ethical care and treatment of organisms and specimens.

**Physical Science Standard II**

The science teacher understands the correct use of tools, materials, equipment and technologies.

**Physical Science Standard III**

The science teacher understands the process of scientific inquiry and its role in science instruction.

**Physical Science Standard IV**

The science teacher has theoretical and practical knowledge about teaching science and about how students learn science.

**Physical Science Standard V**

The science teacher knows the varied and appropriate assessments and assessment practices to monitor science learning.

**Physical Science Standard VI**

The science teacher understands the history and nature of science.

**Physical Science Standard VII**

The science teacher understands how science affects the daily lives of students and how science interacts with and influences personal and societal decisions.

**Physical Science Standard VIII**

The science teacher knows and understands the science content appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in physical science.

**Physical Science Standard XI**

The science teacher knows unifying concepts and processes that are common to all sciences.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## Domains and Competencies

The content covered by this test is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of **competencies**. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

### Domain I – Number Concepts

Competency 001: *The teacher understands the real number system and its structure, operations, algorithms and representations.*

The beginning teacher:

- Understands the concepts of place value, number base and decimal representations of real numbers and rational numbers, including benchmark fractions.
- Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group, ordering of rational and real numbers).
- Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.
- Uses real numbers to model and solve a variety of problems.
- Uses deductive reasoning to simplify and justify algebraic processes.
- Demonstrates how some problems that have no solution in the integer or rational number systems have a solution in the real number system.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



---

Competency 002: *The teacher understands the complex number system and its structure, operations, algorithms and representations.*

The beginning teacher:

- A. Demonstrates how some problems that have no solution in the real number system have a solution in the complex number system.
- B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).
- D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

Competency 003: *The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.*

The beginning teacher:

- A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- B. Applies number theory concepts and principles to justify and prove number relationships.
- C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, noncommutative operations).
- D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.
- F. Uses estimation techniques to solve problems and judge the reasonableness of solutions.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## Domain II – Patterns and Algebra

Competency 004: *The teacher uses patterns to model and solve problems and formulate conjectures.*

The beginning teacher:

- A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.
- B. Uses methods of recursion and iteration to model and solve problems.
- C. Uses the principle of mathematical induction.
- D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).
- F. Determines the validity of logical arguments that include compound conditional statements by constructing truth tables.

Competency 005: *The teacher understands attributes of functions, relations and their graphs.*

The beginning teacher:

- A. Understands when a relation is a function.
- B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.
- E. Applies basic transformations [e.g.,  $k f(x)$ ,  $f(x) + k$ ,  $f(x - k)$ ,  $f(kx)$ ,  $|f(x)|$ ] to a parent function,  $f$ , and describes the effects on the graph of  $y = f(x)$ .
- F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.
- G. Uses graphs of functions to formulate conjectures of identities [e.g.,  $y = x^2 - 1$  and  $y = (x - 1)(x + 1)$ ,  $y = \log x^3$  and  $y = 3 \log x$ ,  $y = \sin(x + \frac{\pi}{2})$  and  $y = \cos x$ ].

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Competency 006: *The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Understands the concept of slope as a rate of change, interprets the meaning of slope and intercept in a variety of situations and understands slope using similar triangles.
- B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and  $y$ -intercept).
- C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems and uses arrays to efficiently manage large collections of data and add, subtract and multiply matrices to solve applied problems, including geometric transformations.
- D. Analyzes the zeros (real and complex) of quadratic functions.
- E. Makes connections between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  representations of a quadratic function and its graph.
- F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

Competency 007: *The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.
- B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.
- C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or differentiable) of a function, the graph of the function and the function's symbolic representation.
- D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.
- E. Analyzes and applies the relationship between inverse variation and rational functions.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- F. Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions, using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.
  - G. Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.
  - H. Models situations using proportional and inverse variations, including describing physical laws such as Hook's law, Newton's second law of motion and Boyle's law.
  - I. Uses precision and accuracy in real-life situations related to measurement and significant figures.
  - J. Applies and analyzes published ratings, weighted averages and indices to make informed decisions.
  - K. Uses proportionality to solve problems involving quantities that are not easily measured.

Competency 008: *The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.
- C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.
- E. Models and solves problems involving exponential growth and decay.
- F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e.,  $f'(x) = k f(x)$ ].

Competency 009: *The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and trigonometric functions.
- B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of trigonometric functions, the graphs of functions and the functions' symbolic representations.
- D. Understands the relationships between trigonometric functions and their inverses and uses those relationships to solve problems.
- E. Uses trigonometric identities to simplify expressions and solve equations.
- F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

Competency 010: *The teacher understands and solves problems using differential and integral calculus.*

The beginning teacher:

- A. Understands the concept of limit and the relationship between limits and continuity.
- B. Relates the concepts of proportionality, rates and average rate of change and applies those concepts to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.
- C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.
  - F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

### **Domain III – Geometry and Measurement**

Competency 011: *The teacher understands measurement as a process.*

The beginning teacher:

- A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.
- D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.
- E. Relates the concept of area under a curve to the limit of a Riemann sum.
- F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane and measurements associated with curves, surfaces and regions in three-space.

Competency 012: *The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.*

The beginning teacher:

- A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.
- C. Applies the properties of parallel and perpendicular lines to solve problems.
- D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.
- E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
  - G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

Competency 013: *The teacher understands the results, uses and applications of Euclidian geometry.*

The beginning teacher:

- A. Analyzes the properties of polygons and their components.
- B. Analyzes the properties of circles and the lines that intersect them.
- C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
- D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
- E. Analyzes cross sections and nets of three-dimensional shapes.
- F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.
- G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life, including in art, architecture and music.
- H. Uses similarity, geometric transformations, symmetry and perspective drawings to describe mathematical patterns and structure in architecture.
- I. Uses scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and nonproportional changes in surface area and volume as applied to fields.
- J. Uses the Pythagorean theorem and special right-triangle relationships to calculate distances.
- K. Uses trigonometric ratios to calculate distances and angle measures as applied to fields, including using models of periodic behavior in art and music.
- L. Solves geometric problems involving indirect measurement, including similar triangles, the Pythagorean theorem, law of sines, law of cosines and the use of dynamic geometry software.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Competency 014: *The teacher understands coordinate, transformational and vector geometry and their connections.*

The beginning teacher:

- A. Identifies transformations (i.e., reflections, translations, glide reflections, rotations and dilations) and explores their properties.
- B. Uses the properties of transformations and their compositions to solve problems.
- C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.
- D. Applies transformations in the coordinate plane.
- E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).
- G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

#### **Domain IV – Probability and Statistics**

Competency 015: *The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.*

The beginning teacher:

- A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.
- B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatterplots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- C. Applies concepts of center, spread, shape and skewness to describe a data distribution.
- D. Understands measures of central tendency (i.e., mean, median and mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



- 
- E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.
  - F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.
  - G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

Competency 016: *The teacher understands concepts and applications of probability.*

The beginning teacher:

- A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.
- B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- C. Determines probabilities by constructing sample spaces to model situations; uses a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results; calculates expected value to analyze mathematical fairness, payoff and risk.
- D. Solves a variety of probability problems using combinations, permutations, and solves problems involving large quantities using combinatorics.
- E. Solves a variety of probability problems using ratios of areas of geometric regions.
- F. Calculates probabilities using the axioms of probability and related theorems and concepts (i.e., addition rule, multiplication rule, conditional probability, independence).
- G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).
- H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 017: *The teacher understands the relationships among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.*

The beginning teacher:

- A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.
  - C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).
  - D. Makes inferences about a population using binomial, normal and geometric distributions.
  - E. Describes, calculates and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
  - F. Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic and power regression models.
  - G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.
  - H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
  - I. Understands the principles of hypotheses testing.
  - J. Determines the number of ways an event may occur using combinations, permutations and the fundamental counting principle.
  - K. Compares theoretical to empirical probability.
  - L. Uses experiments to determine the reasonableness of a theoretical model (i.e., binomial, geometric).
  - M. Identifies limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form.
  - N. Interprets and compares statistical results using appropriate technology given a margin of error.
  - O. Identifies the variables to be used in a study.
  - P. Analyzes possible sources of data variability, including those that can be controlled and those that cannot be controlled.
  - Q. Reports results of statistical studies to a particular audience by selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## Domain V – Mathematical Processes and Perspectives

Competency 018: *The teacher understands mathematical reasoning and problem solving.*

The beginning teacher:

- A. Understands the nature of proof, including indirect proof, in mathematics.
- B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
- D. Uses formal and informal reasoning to justify mathematical ideas.
- E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
- F. Evaluates how well a mathematical model represents a real-world situation.

Competency 019: *The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.*

The beginning teacher:

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).
- C. Translates mathematical ideas between verbal and symbolic forms.
- D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).
- E. Understands the use of visual media (e.g., graphs, tables, diagrams, animations) to communicate mathematical information.
- F. Uses appropriate mathematical terminology to express mathematical ideas.
- G. Explores and applies concepts of financial literacy as it relates to teaching students (e.g., describes the basic purpose of financial institutions, distinguishes the difference between gross income and net income, identifies various savings options, defines different types of taxes, identifies the advantages and disadvantages of different methods of payment).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- H. Applies mathematics to model and solve problems to manage financial resources effectively for lifetime financial security (e.g., distinguishes between fixed and variable expenses, calculates profit in a given situation, develops a system for keeping and using financial records, describes actions that might be taken to balance a budget when expenses exceed income, balances a simple budget).
  - I. Analyzes various voting and selection processes to compare results in given situations, selects and applies an algorithm of interest to solve real-life problems (e.g., using recursion or iteration to calculate population growth or decline, fractals or compound interest; determining validity in recorded and transmitted data using checksums and hashing; evaluating sports rankings, weighted class rankings and search-engine rankings; solving problems involving scheduling or routing using vertex-edge graphs, critical paths, Euler paths or minimal spanning trees), and communicates to peers the application of the algorithm in precise mathematical and nontechnical language.
  - J. Determines or analyzes an appropriate cyclical model for problem situations that can be modeled with periodic functions; determines or analyzes an appropriate piecewise model for problem situations; creates, represents and analyzes mathematical models for various types of income calculations to determine the best option for a given situation; creates, represents and analyzes mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; creates, represents and analyzes mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.

## **Domain VI – Mathematical Learning, Instruction and Assessment**

Competency 020: *The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).*

The beginning teacher:

- A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.
- B. Understands how students differ in their approaches to learning mathematics.
- C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stopwatches, scales, rulers).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- E. Understands how to provide instruction along a continuum from concrete to abstract.
  - F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.
  - G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.
  - H. Understands a variety of questioning strategies to encourage mathematical discourse and help students analyze and evaluate their mathematical thinking.
  - I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

Competency 021: *The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.*

The beginning teacher:

- A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.
- B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.
- D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

## **Domain VII – Scientific Inquiry and Processes**

Competency 022: *The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.*

The beginning teacher:

- A. Uses current sources of information about laboratory safety, including safety regulations and guidelines for the use of science facilities.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- B. Recognizes potential safety hazards in the laboratory and in the field and knows how to apply procedures, including basic first aid, for responding to accidents.
  - C. Employs safe practices in planning, implementing and managing all instructional activities and designs and implements rules and procedures to maintain a safe learning environment.
  - D. Understands procedures for selecting, maintaining and safely using chemicals, tools, technologies, materials, specimens and equipment, including procedures for the recycling, reuse and conservation of laboratory resources and for the safe handling and ethical treatment of organisms.
  - E. Knows how to use appropriate equipment and technology (e.g., Internet, spreadsheet, calculator) for gathering, organizing, displaying and communicating data in a variety of ways (e.g., charts, tables, graphs, diagrams, maps, satellite images, written reports, oral presentations).
  - F. Understands how to use a variety of tools, techniques and technology to gather, organize and analyze data and perform calculations and knows how to apply appropriate methods of statistical measures and analysis.
  - G. Knows how to apply techniques to calibrate measuring devices and understands concepts of precision, accuracy and error with regard to reading and recording numerical data from scientific instruments (e.g., significant figures).
  - H. Uses the International System of Units (i.e., metric system) and performs unit conversions within and across measurement systems.

Competency 023: *The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.*

The beginning teacher:

- A. Understands the nature of science, the relationship between science and technology, the predictive power of science and limitations to the scope of science (i.e., the types of questions that science can and cannot answer).
- B. Knows the characteristics of various types of scientific investigations (e.g., descriptive studies, controlled experiments, comparative data analysis) and how and why scientists use different types of scientific investigations.
- C. Understands principles and procedures for designing and conducting a variety of scientific investigations, with emphasis on inquiry-based investigations, and knows how to communicate and defend scientific results.
- D. Understands how logical reasoning, verifiable observational and experimental evidence and peer review are used in the process of generating and evaluating scientific knowledge.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- E. Understands how to identify potential sources of error in an investigation, evaluate the validity of scientific data and develop and analyze different explanations for a given scientific result.
  - F. Knows the characteristics and general features of systems, how properties and patterns of systems can be described in terms of space, time, energy and matter and how system components and different systems interact.
  - G. Knows how to apply and analyze the systems model (e.g., interacting parts, boundaries, input, output, feedback, subsystems) across the science disciplines.
  - H. Understands how shared themes and concepts (e.g., systems, order and organization; evidence, models and explanation; change, constancy and measurements; evolution and equilibrium; and form and function) provide a unifying framework in science.
  - I. Understands the difference between a theory and a hypothesis, how models are used to represent the natural world and how to evaluate the strengths and limitations of a variety of scientific models (e.g., physical, conceptual, mathematical).

Competency 024: *The teacher understands the history of science, how science impacts the daily lives of students and how science interacts with and influences personal and societal decisions.*

The beginning teacher:

- A. Understands the historical development of science, key events in the history of science and the contributions that diverse cultures and individuals of both genders have made to scientific knowledge.
- B. Knows how to use examples from the history of science to demonstrate the changing nature of scientific theories and knowledge (i.e., that scientific theories and knowledge are always subject to revision in light of new evidence).
- C. Knows that science is a human endeavor influenced by societal, cultural and personal views of the world and that decisions about the use and direction of science are based on factors such as ethical standards, economics and personal and societal biases and needs.
- D. Understands the application of scientific ethics to the conducting, analyzing and publishing of scientific investigations.
- E. Applies scientific principles to analyze factors (e.g., diet, exercise, personal behavior) that influence personal and societal choices concerning fitness and health (e.g., physiological and psychological effects and risks associated with the use of substances and substance abuse).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- F. Applies scientific principles, the theory of probability and risk/benefit analysis to analyze the advantages of, disadvantages of or alternatives to a given decision or course of action.
  - G. Understands the role science can play in helping resolve personal, societal and global issues (e.g., recycling, population growth, disease prevention, resource use, evaluating product claims).

## Domain VIII – Physics

Competency 025: *The teacher understands the description of motion in one and two dimensions.*

The beginning teacher:

- A. Generates, analyzes and interprets graphs describing the motion of a particle.
- B. Applies vector concepts to displacement, velocity and acceleration in order to analyze and describe the motion of a particle.
- C. Solves problems involving uniform and accelerated motion using scalar (e.g., speed) and vector (e.g., velocity) quantities.
- D. Analyzes and solves problems involving projectile motion.
- E. Analyzes and solves problems involving uniform circular and rotary motion.
- F. Understands motion of fluids.
- G. Understands motion in terms of frames of reference and relativity concepts.

Competency 026: *The teacher understands the laws of motion.*

The beginning teacher:

- A. Identifies and analyzes the forces acting in a given situation and constructs a free-body diagram.
- B. Solves problems involving the vector nature of force (e.g., resolving forces into components, analyzing static or dynamic equilibrium of a particle).
- C. Identifies and applies Newton's laws to analyze and solve a variety of practical problems (e.g., properties of frictional forces, acceleration of a particle on an inclined plane, displacement of a mass on a spring, forces on a pendulum).

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



---

Competency 027: *The teacher understands the concepts of gravitational and electromagnetic forces in nature.*

The beginning teacher:

- A. Applies the law of universal gravitation to solve a variety of problems (e.g., determining the gravitational fields of the planets, analyzing properties of satellite orbits).
- B. Calculates electrostatic forces, fields and potentials.
- C. Understands the properties of magnetic materials and the molecular theory of magnetism.
- D. Identifies the source of the magnetic field and calculates the magnetic field for various simple current distributions.
- E. Analyzes the magnetic force on charged particles and current-carrying conductors.
- F. Understands induced electric and magnetic fields and analyzes the relationship between electricity and magnetism.
- G. Understands the electromagnetic spectrum and the production of electromagnetic waves.

Competency 028: *The teacher understands applications of electricity and magnetism.*

The beginning teacher:

- A. Analyzes common examples of electrostatics (e.g., a charged balloon attached to a wall, behavior of an electroscope, charging by induction).
- B. Understands electric current, resistance and resistivity, potential difference, capacitance and electromotive force in conductors and circuits.
- C. Analyzes series and parallel DC circuits in terms of current, resistance, voltage and power.
- D. Identifies basic components and characteristics of AC circuits.
- E. Understands the operation of an electromagnet.
- F. Understands the operation of electric meters, motors, generators and transformers.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Competency 029: *The teacher understands the conservation of energy and momentum.*

The beginning teacher:

- A. Understands the concept of work.
- B. Understands the relationships among work, energy and power.
- C. Solves problems using the conservation of mechanical energy in a physical system (e.g., determining potential energy for conservative forces, conversion of potential to kinetic energy, analyzing the motion of a pendulum).
- D. Applies the work-energy theorem to analyze and solve a variety of practical problems (e.g., finding the speed of an object given its potential energy, determining the work done by frictional forces on a decelerating car).
- E. Understands linear and angular momentum.
- F. Solves a variety of problems (e.g., collisions) using the conservation of linear and angular momentum.

Competency 030: *The teacher understands the laws of thermodynamics.*

The beginning teacher:

- A. Understands methods of heat transfer (i.e., convection, conduction, radiation).
- B. Understands the molecular interpretation of temperature and heat.
- C. Solves problems involving thermal expansion, heat capacity and the relationship between heat and other forms of energy.
- D. Applies the first law of thermodynamics to analyze energy transformations in a variety of everyday situations (e.g., electric light bulb, power-generating plant).
- E. Understands the concept of entropy and its relationship to the second law of thermodynamics.

Competency 031: *The teacher understands the characteristics and behavior of waves.*

The beginning teacher:

- A. Understands interrelationships among wave characteristics such as velocity, frequency, wavelength and amplitude and relates them to properties of sound and light (e.g., pitch, color).
- B. Compares and contrasts transverse and longitudinal waves.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- C. Describes how various waves are propagated through different media.
  - D. Applies properties of reflection and refraction to analyze optical phenomena (e.g., mirrors, lenses, fiber-optic cable).
  - E. Applies principles of wave interference to analyze wave phenomena, including acoustical (e.g., harmonics) and optical phenomena (e.g., patterns created by thin films and diffraction gratings).
  - F. Identifies and interprets how wave characteristics and behaviors are used in medical, industrial and other real-world applications.

Competency 032: *The teacher understands the fundamental concepts of quantum physics.*

The beginning teacher:

- A. Interprets wave-particle duality.
- B. Identifies examples and consequences of the uncertainty principle.
- C. Understands the photoelectric effect.
- D. Understands the quantum model of the atom and can use it to describe and analyze absorption and emission spectra (e.g., line spectra, blackbody radiation) and other phenomenon (e.g., radioactive decay, nuclear forces, nuclear reactions).
- E. Explores real-world applications of quantum phenomena (e.g., lasers, photoelectric sensors, semiconductors, superconductivity).

## **Domain IX – Science Learning, Instruction and Assessment**

Competency 033: *The teacher understands researched-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.*

The beginning teacher:

- A. Knows research-based theories about how students develop scientific understanding and how developmental characteristics, prior knowledge, experience and attitudes of students influence science learning.
- B. Understands the importance of respecting student diversity by planning activities that are inclusive and selecting and adapting science curricula, content, instructional materials and activities to meet the interests, knowledge, understanding, abilities, possible career paths and experiences of all students, including English-language learners.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

- 
- C. Knows how to plan and implement strategies to encourage student self-motivation and engagement in their own learning (e.g., linking inquiry-based investigations to students' prior knowledge, focusing inquiry-based instruction on issues relevant to students, developing instructional materials using situations from students' daily lives, fostering collaboration among students).
  - D. Knows how to use a variety of instructional strategies to ensure all students comprehend content-related texts, including how to locate, retrieve and retain information from a range of texts and technologies.
  - E. Understands the science teacher's role in developing the total school program by planning and implementing science instruction that incorporates schoolwide objectives and the statewide curriculum as defined in the Texas Essential Knowledge and Skills (TEKS).
  - F. Knows how to design and manage the learning environment (e.g., individual, small-group, whole-class settings) to focus and support student inquiries and to provide the time, space and resources for all students to participate in field, laboratory, experimental and nonexperimental scientific investigation.
  - G. Understands the rationale for using active learning and inquiry methods in science instruction and how to model scientific attitudes such as curiosity, openness to new ideas and skepticism.
  - H. Knows principles and procedures for designing and conducting an inquiry-based scientific investigation (e.g., making observations; generating questions; researching and reviewing current knowledge in light of existing evidence; choosing tools to gather and analyze evidence; proposing answers, explanations and predictions; and communicating and defending results).
  - I. Knows how to assist students with generating, refining, focusing and testing scientific questions and hypotheses.
  - J. Knows strategies for assisting students in learning to identify, refine and focus scientific ideas and questions guiding an inquiry-based scientific investigation; to develop, analyze and evaluate different explanations for a given scientific result; and to identify potential sources of error in an inquiry-based scientific investigation.
  - K. Understands how to implement inquiry strategies designed to promote the use of higher-level thinking skills, logical reasoning and scientific problem solving in order to move students from concrete to more abstract understanding.
  - L. Knows how to guide students in making systematic observations and measurements.
  - M. Knows how to sequence learning activities in a way that uncovers common misconceptions, allows students to build upon their prior knowledge and challenges them to expand their understanding of science.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Competency 034: *The teacher knows how to monitor and assess science learning in laboratory, field and classroom settings.*

The beginning teacher:

- A. Knows how to use formal and informal assessments of student performance and products (e.g., projects, laboratory and field journals, rubrics, portfolios, student profiles, checklists) to evaluate student participation in and understanding of inquiry-based scientific investigations.
- B. Understands the relationship between assessment and instruction in the science curriculum (e.g., designing assessments to match learning objectives, using assessment results to inform instructional practice).
- C. Knows the importance of monitoring and assessing students' understanding of science concepts and skills on an ongoing basis by using a variety of appropriate assessment methods (e.g., performance assessment, self-assessment, peer assessment, formal/informal assessment).
- D. Understands the purposes, characteristics and uses of various types of assessment in science, including formative and summative assessments, and the importance of limiting the use of an assessment to its intended purpose.
- E. Understands strategies for assessing students' prior knowledge and misconceptions about science and how to use those assessments to develop effective ways to address the misconceptions.
- F. Understands characteristics of assessments, such as reliability, validity and the absence of bias in order to evaluate assessment instruments and their results.
- G. Understands the role of assessment as a learning experience for students and strategies for engaging students in meaningful self-assessment.
- H. Recognizes the importance of selecting assessment instruments and methods that provide all students with adequate opportunities to demonstrate their achievements.
- I. Recognizes the importance of clarifying teacher expectations by sharing evaluation criteria and assessment results with students.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## Approaches to Answering Multiple-Choice Questions

The purpose of this section is to describe multiple-choice question formats that you will typically see on the Physics/Mathematics 7–12 test and to suggest possible ways to approach thinking about and answering them. These approaches are intended to supplement and complement familiar test-taking strategies with which you may already be comfortable and that work for you. Fundamentally, the most important component in assuring your success on the test is knowing the content described in the test framework. This content has been carefully selected to align with the knowledge required to begin a career as a Physics/Mathematics 7–12 teacher.

The Physics/Mathematics 7–12 test is designed to include a total of 120 multiple-choice questions, out of which 100 are scored. These multiple-choice questions are designed to assess your knowledge of the content described in the test framework. In most cases, you are expected to demonstrate more than just your ability to recall factual information. You may be asked to think critically about the information, to analyze it, consider it carefully, compare it with other knowledge you have or make a judgment about it.

The number of scored questions will not vary; however, the number of questions that are not scored may vary in the actual test. Your final scaled score will be based only on scored questions. The questions that are not scored are being pilot tested to collect information about how these questions will perform under actual testing conditions. These pilot questions are not identified on the test.

Leave no questions unanswered. Your score will be determined by the number of questions you answer correctly.

**Calculators.** Some questions on the Physics/Mathematics 7–12 test are designed to be solved with a graphing calculator. An online graphing calculator will be part of the testing software for this test. You can find a tutorial for the online calculator at [www.texas.ets.org/texas/acttutorial](http://www.texas.ets.org/texas/acttutorial).

**Definitions and Formulas.** A set of definitions and formulas will be provided as part of the test. A copy of those definitions and formulas is provided in this preparation manual.

**Definitions and Physical Constants.** A set of definitions and physical constants will be provided as part of the test. A copy of those definitions and physical constants is provided in this preparation manual.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

## How to Approach Unfamiliar Question Formats

Some questions include introductory information such as a map, table, graph or reading passage (often called a stimulus) that provides the information the question asks for. New formats for presenting information are developed from time to time. Tests may include audio and video stimulus materials such as a movie clip or some kind of animation, instead of a map or reading passage. Other tests may allow you to zoom in on the details in a graphic or picture.

Tests may also include interactive types of questions. These questions take advantage of technology to assess knowledge and skills that go beyond what can be assessed using standard single-selection multiple-choice questions. If you see a format you are not familiar with, **read the directions carefully**. The directions always give clear instructions on how you are expected to respond.

For most questions, you will respond by clicking an oval to choose a single answer choice from a list of options. Other questions may ask you to respond by:

- **Selecting all that apply.** In some questions, you will be asked to choose all the options that answer the question correctly.
- **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one choice within a set of answers can be selected.
- **Clicking parts of a graphic.** In some questions, you will choose your answer by clicking on location(s) on a graphic such as a map or chart, as opposed to choosing from a list.
- **Clicking on sentences.** In questions with reading passages, you may be asked to choose your answer by clicking on a sentence or sentences within the reading passage.
- **Dragging and dropping answer choices into “targets” on the screen.** You may be asked to choose an answer from a list and drag it into the appropriate location in a table, paragraph of text or graphic.
- **Selecting options from a drop-down menu.** This type of question will ask you to select the appropriate answer or answers by selecting options from a drop-down menu (e.g., to complete a sentence).

Remember that with every question, you will get clear instructions on how to respond.

NOTE: After clicking on a link, right click and select “Previous View” to go back to original text.

---

## Question Formats

You may see the following types of multiple-choice questions on the test:

- Single Questions
- Clustered Questions

On the following pages, you will find descriptions of these commonly used question formats, along with suggested approaches for responding to each type.

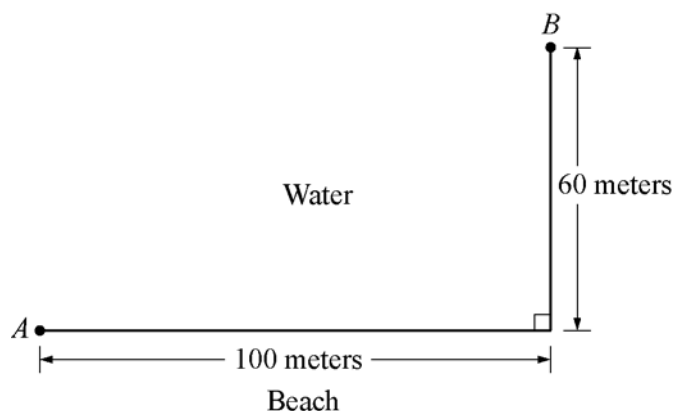
## Single Questions

The single-question format presents a direct question or an incomplete statement. It can also include a reading passage, graphic, table or a combination of these. Four or more answer options appear below the question.

The following question is an example of the single-question format. It tests knowledge of Physics/Mathematics 7–12 Competency 010: *The teacher understands and solves problems using differential and integral calculus.*

### Example

**Use the diagram below to answer the question that follows.**



A lifeguard sitting on a beach at point  $A$  sees a swimmer in distress at point  $B$ . The lifeguard can run at a rate of 3 meters per second and can swim at a rate of 1.5 meters per second. To minimize the amount of time it takes to reach the swimmer, how far along the beach should the lifeguard run before entering the water?

- A. 40 meters
- B. 65 meters
- C. 73 meters
- D. 100 meters

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

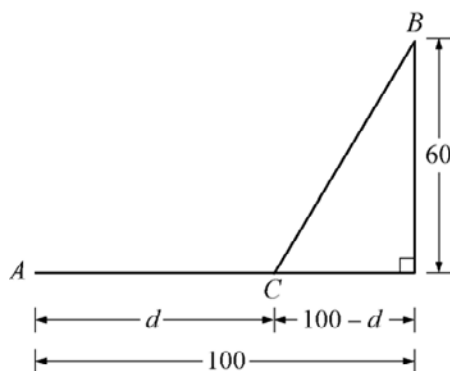


---

## Suggested Approach

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice and mark it on your answer sheet.

In analyzing this problem, redrawing the diagram to highlight the important information may be helpful.



Let  $d$  represent the distance in meters that the lifeguard runs along the beach. Then by an application of the Pythagorean theorem, the distance traveled in water is represented by  $\sqrt{60^2 + (100 - d)^2}$ .

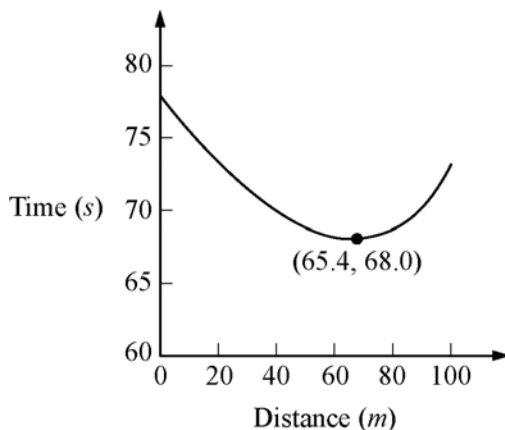
Because  $distance = rate \times time$  and the lifeguard can run at 3 meters per second and swim at 1.5 meters per second, the time it takes the lifeguard to run along the beach,  $t_b$ , can be represented by  $\frac{d}{3}$ , and the time it takes the lifeguard to swim in the water,  $t_w$ , can be represented by  $\frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$ . Thus, the total time,  $t$ , it takes the lifeguard to travel to the swimmer can be represented by  $t_b + t_w$ .

To solve the problem, we need to find the value of  $d$  that minimizes the function

$$t = t_b + t_w = \frac{d}{3} + \frac{\sqrt{60^2 + (100 - d)^2}}{1.5}.$$

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

This can be done using either differential calculus or a graphing approach. We will use a graphing approach. A graphing calculator can be used to produce a graph similar to the one below.



Using the capabilities of the calculator, the minimum value of the function  $t$  occurs when  $d$  is approximately 65 meters, or option B. Option A results from dividing 60 by 1.5, which is the time required to swim 60 meters. Option C results from misusing parentheses when entering the equation for  $t$  into the graphing utility;

i.e., entering  $\sqrt{\frac{60^2 + (100 - d)^2}{1.5}}$  instead of option D results from minimizing the

function  $t_w = \frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$  instead of the expression for  $t$ , the total time required to reach the swimmer. The correct response is **option B**.

### Clustered Questions

Clustered questions are made up of a stimulus and two or more questions relating to the stimulus. The stimulus material can be a reading passage, description of an experiment, graphic, table or any other information necessary to answer the questions that follow.

You can use several different approaches to respond to clustered questions. Some commonly used strategies are listed below.

**Strategy 1** Skim the stimulus material to understand its purpose, its arrangement and/or its content. Then read the questions and refer again to the stimulus material to obtain the specific information you need to answer the questions.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

**Strategy 2** Read the questions *before* considering the stimulus material. The theory behind this strategy is that the content of the questions will help you identify the purpose of the stimulus material and locate the information you need to answer the questions.

**Strategy 3** Use a combination of both strategies. Apply the “read the stimulus first” strategy with shorter, more familiar stimuli and the “read the questions first” strategy with longer, more complex or less familiar stimuli. You can experiment with the sample questions in this manual and then use the strategy with which you are most comfortable when you take the actual test.

Whether you read the stimulus before or after you read the questions, you should read it carefully and critically. You may want to note its important points to help you answer the questions.

As you consider questions set in educational contexts, try to enter into the identified teacher’s frame of mind and use that teacher’s point of view to answer the questions that accompany the stimulus. Be sure to consider the questions only in terms of the information provided in the stimulus — not in terms of your own experiences or individuals you may have known.

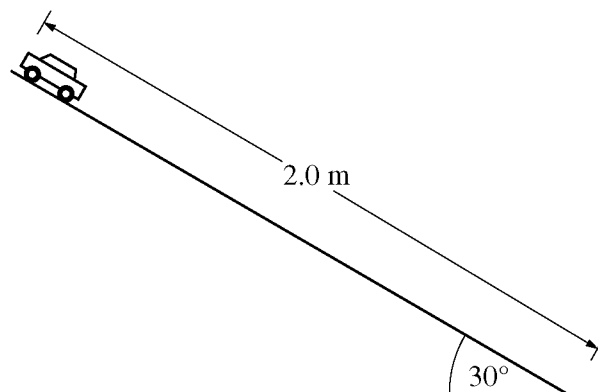
### Example 1

First read the stimulus (a description of a physics experiment along with a data table).

**Use the information below to answer the two questions that follow.**

A group of students is measuring how long it takes a toy car released from rest to roll down a straight inclined track. The data from the experiment are summarized below.

Mass of car	0.10 kg
Length of incline	2.0 m
Slope of incline	$30^\circ$
Average time	1.2 s



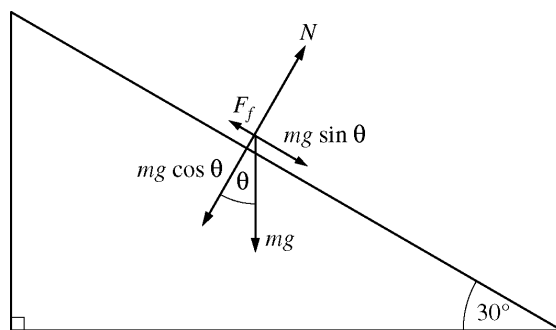
NOTE: After clicking on a link, right click and select “Previous View” to go back to original text.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Physics/Mathematics 7–12 Competency 026: *The teacher understands the laws of motion.*

1. What is the magnitude of the gravitational force acting on the car in the direction of the toy car’s motion down the track?
  - A. 0.10 N
  - B. 0.49 N
  - C. 0.85 N
  - D. 0.98 N

### Suggested Approach

The first step is to identify the forces acting on the car. In this case the forces acting on the car are the force of gravity, the force of friction and the normal force from the inclined plane on the car. The next step is to draw a free body diagram showing these forces resolved into their appropriate components.



To determine the magnitude of the gravitational force acting on the car in the direction of the car’s motion down the track, it is necessary to determine the component of the gravitational force along the incline. For an inclined plane, this component is given by  $F = mg \sin \theta$ , where  $m$  is the mass of the car,  $g$  is the acceleration due to gravity ( $9.8 \text{ m/s}^2$ ), and  $\sin \theta$  is the sine of the angle of the incline with the horizontal. Substituting the given values into the expression and using the fact that  $\sin 30^\circ = 0.5$  results in the numerical value for the force component acting along the plane, or  $F = 0.49 \text{ N}$ . This is option B.

Option A is the mass of the car and is therefore incorrect. Option C results from incorrectly using  $mg \cos 30^\circ$  for the component of the gravitational force in the direction of the car’s motion. Option D is the weight of the car, which is equal to the magnitude of the gravitational force  $mg$  toward the center of the earth. The correct response is **option B**.

Now you are ready to answer the next question. The second question also measures Competency 026: *The teacher understands the laws of motion.*

NOTE: After clicking on a link, right click and select “Previous View” to go back to original text.

- 
2. Assuming the acceleration of the car down the track is constant, what is the net force acting on the car in the direction of the car's motion down the track?
- A. 0.21 N
  - B. 0.28 N
  - C. 0.56 N
  - D. 0.98 N

### Suggested Approach

The second question for this stimulus asks for the net force acting on the car in the direction of the car's motion. According to Newton's second law of motion, the net force on any object in the direction of the object's motion is equal to the object's mass multiplied by its acceleration, or  $F_{\text{net}} = ma$ . Because the mass of the car is known, it is necessary to find the acceleration of the car. The question tells us to assume the acceleration is constant. Also, it is given from the original stimulus data that the car starts from rest and travels a distance of 2.0 m in 1.2 s. The expression for the distance traveled by an object undergoing constant acceleration,  $x = \frac{1}{2}at^2 + v_0t + x_0$  simplifies to  $x = \frac{1}{2}at^2$ . In this problem, therefore, solving for  $a$  yields  $a = \frac{2x}{t^2} = \frac{2(2.0)}{(1.2)^2} = 2.8 \text{ m/s}^2$ . Multiplying this value by the mass of the car results in 0.28 N, which is option B.

Option A results from incorrectly calculating the acceleration as the distance the object travels divided by the time required, or  $\frac{2.0}{1.2}$ , and using this value to find the force. Option C results from correctly determining the acceleration and multiplying the result by the mass of the car but then incorrectly trying to find the component of the force parallel to the plane by dividing the result by  $\sin 30^\circ$ , or 0.5. Option D is the force of gravity on the object. The correct response is **option B**.

### Example 2

First read the stimulus (a learning expectation from the statewide mathematics curriculum).

**Use the student expectation below from the Texas Essential Knowledge and Skills (TEKS) to answer the two questions that follow.**

The student uses characteristics of the quadratic parent function to sketch the related graphs and makes connections between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  symbolic representations of quadratic functions.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Physics/Mathematics 7–12, Competency 020: *The teacher understands how children learn mathematics, and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).*

1. Which of the following algebraic techniques will students need to know to symbolically convert a quadratic function of the form  $y = ax^2 + bx + c$  into the form  $y = a(x - h)^2 + k$  ?
- A. Solving systems of equations
  - B. Completing the square
  - C. Solving quadratic equations
  - D. Simplifying polynomial expressions

### Suggested Approach

You are asked to identify the algebraic technique that students should use to convert the expression  $y = ax^2 + bx + c$  into the expression  $y = a(x - h)^2 + k$ . The following steps show how this conversion can be achieved.

First rewrite the expression  $y = ax^2 + bx + c$  as  $y = a\left(x^2 + \frac{b}{a}x\right) + c$  by factoring  $a$  from the quantity  $ax^2 + bx$ . Next, take one-half the coefficient of the linear term, square it, and add this quantity inside the parentheses while adding the product of the quantity's additive inverse and  $a$  outside of the parentheses. Note that this is equivalent to adding  $\frac{b^2}{4a}$  and  $-\frac{b^2}{4a}$  to the same side of the equation as follows:

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

Notice that the quantity inside the parentheses is a perfect square and can be factored.

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

This expression is equivalent to  $y = a(x - h)^2 + k$ , with  $h = -\frac{b}{2a}$  and

$k = \frac{4ac - b^2}{4a}$ , which are the  $x$ - and  $y$ -coordinates of the vertex of the graph of  $y = ax^2 + bx + c$ . This algebraic method of converting the first expression into the second is known as completing the square. Therefore, **option B is correct.**

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

Option A, solving systems of equations, is not helpful in this situation because the student is being asked to rewrite an equation, not solve it. Option C is incorrect because the student is being asked to rewrite a quadratic equation, not solve it.

Finally, although one can simplify the expression  $y = a(x - h)^2 + k$  and compare it to  $y = ax^2 + bx + c$ , this approach is ineffective when applied in the opposite direction, which makes option D incorrect.

Now you are ready to answer the next question. The second question measures Competency 021: *The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.*

2. Which of the following exercises best assesses student understanding of the expectation from the statewide curriculum (TEKS)?
- A. Use a graphing calculator to graph the function  $y = x^2 - 4x + 3$ , and use the graph to find the zeros of the function.
  - B. Write a real-world word problem that is modeled by the function  $y = x^2 - 4x + 3$ , and relate the solution of the function to the graph of  $y = x^2 - 4x + 3$ .
  - C. Describe how the graph of  $y = (x - 3)(x - 1)$  is related to the graph of  $y = x^2 - 4x + 3$ .
  - D. Describe how the graph of  $y = x^2$  is related to the graph of  $y = x^2 - 4x + 3$ .

### Suggested Approach

You are asked to select an activity that would best assess student understanding of converting a function of the form  $y = ax^2 + bx + c$  into the form

$y = a(x - h)^2 + k$  and then analyzing the graph of this function in relation to the quadratic parent function  $y = x^2$ . Carefully read each of the responses to determine how well they assess student understanding of this topic.

Option A asks the student to enter a quadratic function into a graphing calculator and then use the capabilities of the graphing calculator to estimate the zeros of the function. This is a method of using technology to solve a quadratic equation, and hence is incorrect.

Option B asks the student to create a problem that can be modeled by a specific quadratic equation and to relate the graph of the equation to the problem. This assessment would be useful for evaluating student understanding of applications of quadratic functions, but not for assessing understanding of the two different

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

symbolic representations of the quadratic function. Option B is therefore incorrect. Option C assesses understanding of the fact that a factored quadratic function has the same graph as the expanded, or unfactored, quadratic function. Option C would not assess the given learning expectation and is therefore incorrect.

Option D assesses student understanding of how the graph of  $y = x^2$  is related to that of a more complicated quadratic function involving a linear term and a constant term. Expressing the function  $y = x^2 - 4x + 3$  in the form  $y = (x - 2)^2 - 1$  allows a student to determine by inspection that the vertex is at  $(2, -1)$ . This implies that the graph of  $y = x^2 - 4x + 3$  can be obtained by translating the graph of  $y = x^2$  two units in the positive  $x$ -direction and one unit in the negative  $y$ -direction.

This analysis of the four choices should lead you to select **option D as the correct response**.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



---

## Multiple-Choice Practice Questions

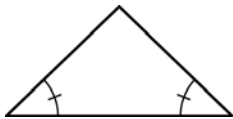


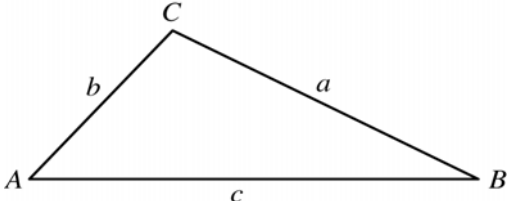
This section presents some sample test questions for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample question is accompanied by the competency that it measures. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual test.

For each sample test question, there is a correct answer and a rationale for each answer option. Please note that the sample questions are not necessarily presented in competency order.

The sample questions are included to illustrate the formats and types of questions you will see on the test; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual test.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

## Definitions and Formulas for Mathematics 7–12

<p style="text-align: center;"><b>CALCULUS</b></p> <p><b>First Derivative:</b> <math>f'(x) = \frac{dy}{dx}</math></p> <p><b>Second Derivative:</b> <math>f''(x) = \frac{d^2y}{dx^2}</math></p> <p style="text-align: center;"><b>PROBABILITY</b></p> <p><math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math></p> <p><math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math></p>	<p style="text-align: center;"><b>ALGEBRA</b></p> <p><b>i</b> <math>i^2 = -1</math></p> <p><math>A^{-1}</math> inverse of matrix A</p> <p><math>A = P\left(1 + \frac{r}{n}\right)^{nt}</math> Compound interest, where A is the final value P is the principal r is the interest rate t is the term n is the number of divisions within the term</p> <p><math>[x] = n</math> Greatest integer function, where n is the integer such that <math>n \leq x &lt; n + 1</math></p>
<p style="text-align: center;"><b>GEOMETRY</b></p> <p><b>Congruent Angles</b></p>  <p><b>Congruent Sides</b></p>  <p><b>Parallel Sides</b></p>  <p><b>Circumference of a Circle</b></p> <p style="text-align: center;"><math>C = 2\pi r</math></p>	<p style="text-align: center;"><b>VOLUME</b></p> <p><b>Cylinder:</b> (area of base) <math>\times</math> height</p> <p><b>Cone:</b> <math>\frac{1}{3}</math> (area of base) <math>\times</math> height</p> <p><b>Sphere:</b> <math>\frac{4}{3}\pi r^3</math></p> <p><b>Prism:</b> (area of base) <math>\times</math> height</p> <p style="text-align: center;"><b>AREA</b></p> <p><b>Triangle:</b> <math>\frac{1}{2}</math> (base <math>\times</math> height)</p> <p><b>Rhombus:</b> <math>\frac{1}{2}</math> (diagonal<sub>1</sub> <math>\times</math> diagonal<sub>2</sub>)</p> <p><b>Trapezoid:</b> <math>\frac{1}{2}</math> height (base<sub>1</sub> + base<sub>2</sub>)</p> <p><b>Sphere:</b> <math>4\pi r^2</math></p> <p><b>Circle:</b> <math>\pi r^2</math></p> <p><b>Lateral surface area of cylinder:</b> <math>2\pi rh</math></p>
<p style="text-align: center;"><b>TRIGONOMETRY</b></p> <p><b>Law of Sines:</b> <math>\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}</math></p> <p><math>c^2 = a^2 + b^2 - 2ab \cos C</math></p> <p><b>Law of Cosines:</b> <math>b^2 = a^2 + c^2 - 2ac \cos B</math></p> <p><math>a^2 = b^2 + c^2 - 2bc \cos A</math></p>	

### End of Definitions and Formulas

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

### ***Definitions and Physical Constants for Physics 7–12***

---

The value of  $9.8 \text{ m/s}^2$  is used for the acceleration of gravity near Earth's surface.

The universal gas constant is  $8.314 \text{ J/K-mol}$  or  $0.08206 \text{ L-atm/K-mol}$ .

Planck's constant is  $6.6256 \times 10^{-34} \text{ J-s}$ .

Avogadro's number is  $6.022 \times 10^{23}$ .

The right-hand rule is used with conventional current (the flow of positive charge from the positive terminal to the negative terminal).

---

### ***End of Definitions and Physical Constants***

---

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

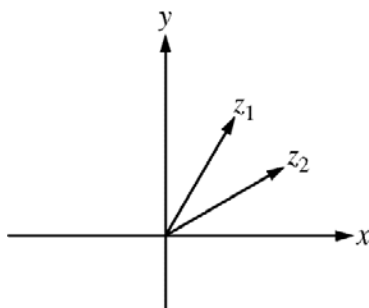
COMPETENCY 001

1. A light year is the distance light travels in one year. The star Procyon is approximately  $1.08 \times 10^{17}$  meters from our Sun. If light travels at  $3.00 \times 10^8$  meters per second, and if there are approximately  $3.16 \times 10^7$  seconds in a year, what is the approximate distance, in light years, from Procyon to our Sun?
- A. 11.39
  - B. 87.78
  - C.  $1.14 \times 10^2$
  - D.  $8.78 \times 10^{-2}$

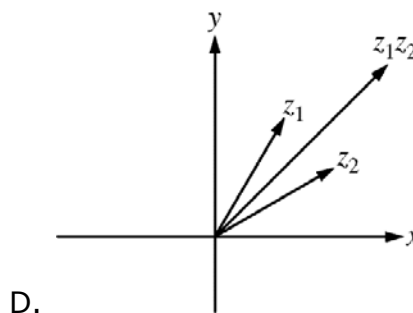
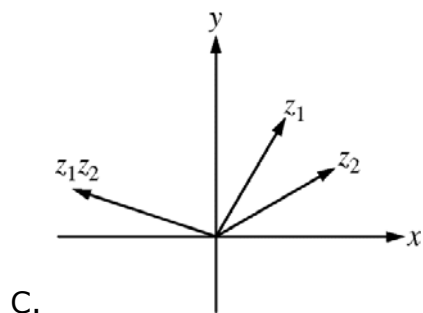
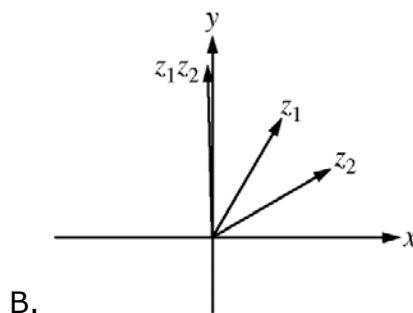
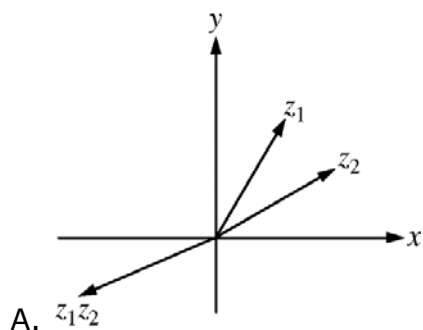
Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the figure below to answer the question that follows.



2. Two complex numbers,  $z_1$  and  $z_2$  are shown in the complex plane above. Which of the following graphs depicts the product  $z_1 z_2$  in the complex plane?



Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 003

3. Given that  $n - 5$  and  $n + 6$  are positive integers both divisible by prime number  $p$ , which of the following integers must also be divisible by  $p$ ?
- A.  $n + 1$
  - B.  $n - 30$
  - C.  $n + 28$
  - D.  $n - 11$

Answer and Rationale

COMPETENCY 003

4. On the first day of 2011 Asad deposited \$500 into a savings account earning 3 percent annual interest, compounded annually at the end of the year. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2012, Asad deposited an additional \$500 into the account, earning the same annual interest, compounded annually. Asad made no additional deposits or withdrawals from the account during the year. On the first day of 2013, Asad deposited an additional \$500 into the account, earning the same annual interest, compounded annually. If no additional deposits or withdrawals were made, which of the following expressions represents the amount of money in Asad's account at the end of 2013?
- A.  $3[500 + 500(0.03)] + 500(0.06)$
  - B.  $3[500 + 500(1.03) + 500(2.06)]$
  - C.  $500(1.03) + 500(1.03)^2 + 500(1.03)^3$
  - D.  $500(0.03) + 500(0.03)^2 + 500(0.03)^3$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 008

5. Which of the following is a vertical asymptote of the graph of  $y = 3 + \log_2(x - 4)$ ?
- A.  $y = 3$
  - B.  $y = -4$
  - C.  $x = -4$
  - D.  $x = 4$

Answer and Rationale

COMPETENCY 007

**Use the equations below to answer the question that follows.**

$$y = -3x - 2$$

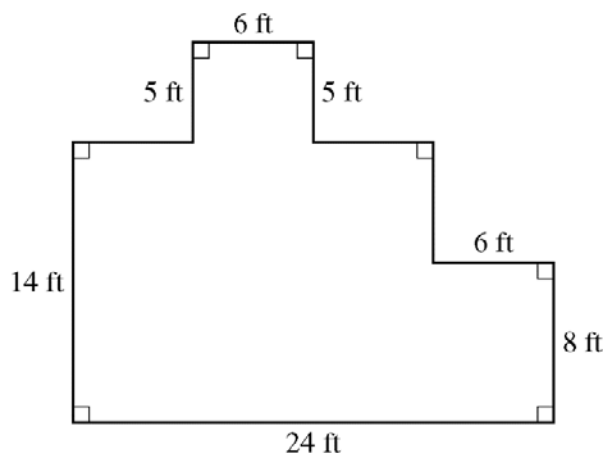
$$y = \begin{cases} x + 2, & x \leq 2 \\ 10 - 2x, & x > 2 \end{cases}$$

6. Which of the following are the points of intersection of the graphs in the  $xy$ -plane of the equations shown?
- A.  $(-1, 1)$  only
  - B.  $(-1, -5)$  only
  - C.  $(-12, 34)$  only
  - D.  $(-1, 1)$  and  $(-12, 34)$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the figure below to answer the question that follows.



7. The floor plan of Nick's den is shown. Nick wishes to purchase square tiles that are 12 inches on each side to tile the entire floor. To allow for possible breakage, he plans to purchase more tiles to cover an additional 10 percent of the floor area. How many tiles will Nick purchase?
- A. 330 tiles
  - B. 363 tiles
  - C. 366 tiles
  - D. 402 tiles

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



---

COMPETENCY 014

8. Let the vectors  $i$  and  $j$  be given by  $i = (1,0)$  and  $j = (0,1)$ . A person walks 80 feet due east, and then turns counter-clockwise  $40^\circ$  and walks an additional 60 feet in that direction. Which of the following vectors describes the person's final position relative to his or her starting position?
- A.  $(60 \cos 40^\circ)i + (80 + 60 \cos 40^\circ)j$
- B.  $(60 \cos 40^\circ)i + (80 + 60 \sin 40^\circ)j$
- C.  $(80 + 60 \sin 40^\circ)i + (60 \sin 40^\circ)j$
- D.  $(80 + 60 \cos 40^\circ)i + (60 \sin 40^\circ)j$

Answer and Rationale

COMPETENCY 011

9. Which of the following definite integrals represents the length of the curve  $y = x^2 + 1$  between the points  $(0, 1)$  and  $(2, 5)$ ?
- A.  $\int_0^2 (1 + 4x^2) dx$
- B.  $\int_0^2 \sqrt{1 + 4x^2} dx$
- C.  $\int_0^2 \sqrt{1 + x^2} dx$
- D.  $\pi \int_0^2 4x^2 dx$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 016

10. Mr. Allen’s math class consists of 15 boys and 15 girls. Each day he randomly selects 3 students to present homework problems on the blackboard. What is the probability that the three chosen students will be all boys?

(Note:  ${}_nC_r = \frac{n!}{(n-r)!r!}$  denotes combinations of  $n$  objects taken  $r$  at a time,

and  ${}_nP_r = \frac{n!}{(n-r)!}$  denotes permutations of  $n$  objects taken  $r$  at a time.)

- A.  $\frac{{}_{15}P_3}{{}_{30}C_3}$
- B.  $\frac{{}_{15}C_3}{{}_{15}P_3}$
- C.  $\frac{{}_{15}C_3}{{}_{30}C_3}$
- D.  $\frac{{}_{15}P_3}{{}_{15}C_3}$

Answer and Rationale

COMPETENCY 017

11. A packing company packages approximately 10,000 packages of noodles each day. The packages have a mean weight of 16 ounces with a standard deviation of 0.2 ounces. An inspector selects multiple samples of 25 packages each and computes the mean weight of each sample. Which of the following is the best estimate of the standard deviation of the mean weights of the samples?

- A. 0.02
- B. 0.04
- C. 0.2
- D. 1.0

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 019

12. Let  $C$  be the set of all right-circular cylinders of radius 3. Let  $v(x)$  be the volume of the cylinders in  $C$  as a function of height,  $x \geq 0$ . The graph of the function  $y = v(x)$  in the  $xy$ -plane is
- A. a horizontal line through the point  $(0, 9\pi)$ .
  - B. a line with slope  $9\pi$  and  $y$ -intercept 0.
  - C. a parabola with vertex  $(0, 0)$  passing through the point  $(1, 9\pi)$ .
  - D. a cubic polynomial passing through the points  $(0, 0)$ ,  $(1, 9\pi)$ , and  $(3, 27\pi)$ .

Answer and Rationale

COMPETENCY 018

13. Mr. Matthews' precalculus class is studying even and odd functions. Mr. Matthews gave a function to a group of students in his class and asked them to investigate its symmetry without the use of a graphing calculator. Katie, Shayan and Diego are given the function  $f(x) = \frac{x^3 + x}{|x| - 4}$ , where  $x \neq \pm 4$ .
- Katie notes that  $f(1) = -\frac{2}{3}$  and  $f(-1) = \frac{2}{3}$ . Shayan computes  $f(3) = -30$  and  $f(-3) = 30$ . Diego discovers that  $f(2) = -5$  and  $f(-2) = 5$ . After comparing notes, the three students conjecture that the function is odd because the graph exhibits symmetry with respect to the origin. Which of the following activities would enable the group to verify this conjecture?
- A. The students could look for a counterexample by trying several values for  $x$ . If they are not able to find a counterexample, they can deduce that the function exhibits symmetry with respect to the origin and is therefore odd.
  - B. The students could evaluate the function at an additional eight pairs of values of  $x$  that are additive inverses and note that in each case,  $f(-x) = -f(x)$ . They could then deduce that the conjecture is true.
  - C. The students could simplify the algebraic expression for  $f(-x)$  and establish that it is equal to  $-f(x)$  for all values of  $x$ ,  $x \neq \pm 4$ .
  - D. The students could simplify the algebraic expression for  $f(-x)$  and establish that it is equal to  $f(x)$  for all values of  $x$ ,  $x \neq \pm 4$ .

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

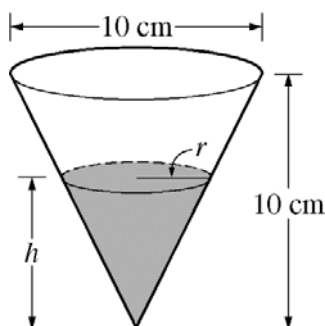
COMPETENCY 021

14. Which of the following is a main purpose of formative assessments?
- A. To provide feedback for teachers and students to improve the teaching and learning activities used for instruction
  - B. To summarize and record overall achievement at the end of an instructional unit
  - C. To evaluate the effectiveness of the curriculum
  - D. To identify students ready for promotion to the next level of a course

Answer and Rationale

COMPETENCY 011

**Use the figure below to answer the question that follows.**



15. An inverted circular cone contains water, as shown. If the height,  $h$ , of the water is 8 centimeters, what is the radius,  $r$ , of the surface of the water, in centimeters?
- A. 8
  - B. 6
  - C. 4
  - D. 2

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

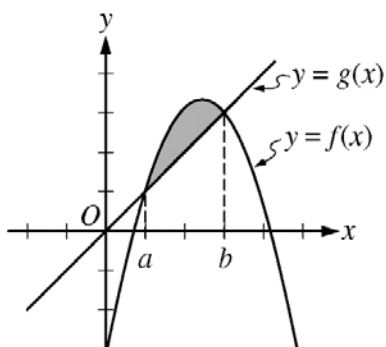
COMPETENCY 001

16. The number of bacteria in a colony increases from 400 to 800 during 1 hour. What is the rate of growth?
- A. 400% per hour
  - B. 200% per hour
  - C. 100% per hour
  - D. 50% per hour

Answer and Rationale

COMPETENCY 011

**Use the figure below to answer the question that follows.**



17. Let  $f$  and  $g$  be continuous functions defined on the interval  $[a, b]$ , as shown. Which of the following can be used to find the area of the shaded region?

- A.  $\int_a^b f(x)dx - \int_a^b g(x)dx$
- B.  $\int_a^b g(x)dx - \int_b^a f(x)dx$
- C.  $\int_a^b \pi(f(x) - g(x))^2 dx$
- D.  $2\pi \int_a^b x(f(x) - g(x)) dx$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 010

18.  $\frac{d}{dx} \int_0^x \sin t \cos t \, dt =$

- A.  $\frac{\sin^2 x \cos^2 x}{4}$
- B.  $\cos 2x$
- C.  $\frac{\sin^2 x}{2}$
- D.  $\sin x \cos x$

Answer and Rationale

COMPETENCY 019

19. Bank A and Bank B each offer 5 percent interest rate on savings accounts, but Bank A compounds the interest annually and Bank B compounds the interest semiannually. If Samantha invests \$10,000 in a savings account in Bank B, and makes no other transactions in her account, how much more interest will she earn in a year than she would have earned if she had invested the money in Bank A?
- A. \$ 5.00
  - B. \$ 6.25
  - C. \$ 7.25
  - D. \$ 8.50

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

COMPETENCY 015

Use the chart below to answer the question that follows.

SALARIES OF STUDENTS WORKING PART-TIME



20. The bar graph shown represents the distributions of weekly salaries, rounded to the nearest dollar, for a group of students with part-time jobs. Which of the following statements is true?
- A. The range of the weekly salaries is \$499.
  - B. The median weekly salary is between \$240 and \$260.
  - C. The average weekly salary is between \$167 and \$267.
  - D. The mode of the weekly salaries is between \$200 and \$299.

Answer and Rationale

COMPETENCY 022

21. The standard deviation of a set of data measurements is related to which of the following? Select all that apply.
- A. Precision
  - B. Measurement repeatability
  - C. Systematic error
  - D. Reproducibility

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 023

22. Scientific theories are based on which of the following? Select all that apply.

- A. Observations
- B. Experiments
- C. Empirical generalizations
- D. Personal beliefs and speculations

Answer and Rationale

COMPETENCY 025

23. A ball of mass  $m$  is thrown horizontally with an initial speed  $v_0$  from the top of a building that is height  $h$  above level ground. In the absence of air resistance, what is the speed of the ball when it strikes the ground?

- A.  $v_0$
- B.  $\sqrt{2gh}$
- C.  $v_0 + \sqrt{2gh}$
- D.  $\sqrt{v_0^2 + 2gh}$

Answer and Rationale

COMPETENCY 025

24. Two satellites are each in a circular orbit around Earth at distances  $R$  and  $2R$ , respectively, from Earth's center. If the satellite at distance  $R$  has an orbital period equal to  $T$ , the satellite at distance  $2R$  must have an orbital period equal to

- A.  $2\sqrt{2}T$
- B.  $2T$
- C.  $T$
- D.  $\frac{1}{2}T$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



---

COMPETENCY 025

25. A block of mass 2 kg and volume  $4 \times 10^{-4} \text{ m}^3$  is suspended from a thin wire and completely submerged in water. What is the tension in the wire? (Assume  $g = 10 \text{ m/s}^2$  .)

- A. 0
- B. 4 N
- C. 16 N
- D. 20 N

Answer and Rationale

COMPETENCY 026

26. A 40 kg box is pulled up a long, frictionless ramp by a 400 N force. If the ramp is inclined at an angle of  $30^\circ$  with respect to the horizontal, what is the magnitude of the acceleration of the block? (Assume  $g = 10 \text{ m/s}^2$  .)

- A. 0
- B.  $5 \text{ m/s}^2$
- C.  $10 \text{ m/s}^2$
- D.  $15 \text{ m/s}^2$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 026

27. A 50 N force is being applied to a 10 kg block on a rough horizontal surface. If the coefficient of kinetic friction between the moving block and the surface is 0.25, what is the magnitude of the acceleration of the block? (Assume  $g = 10 \text{ m/s}^2$ .)
- A.  $2.5 \text{ m/s}^2$
  - B.  $5 \text{ m/s}^2$
  - C.  $7.5 \text{ m/s}^2$
  - D.  $10 \text{ m/s}^2$

Answer and Rationale

COMPETENCY 027

28. A simple pendulum of length  $L$  has a period  $T$  on Earth. If a simple pendulum is to have the same period  $T$  on Jupiter, where the gravitational force is 2.5 times as great as the gravitational force on Earth, what does the length of the pendulum have to be?
- A.  $2.5L$
  - B.  $\sqrt{2.5}L$
  - C.  $L$
  - D.  $\frac{L}{2.5}$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 027

29. A planet moves in an elliptical orbit about a star. Which of the following is true about the orbital speed of the planet?
- A. It is constant.
  - B. It varies and is greatest when the planet is closest to the star.
  - C. It varies and is greatest when the planet is farthest from the star.
  - D. It cannot be determined from known physical laws.

Answer and Rationale

COMPETENCY 027

30. A coil of wire produces a magnetic field of magnitude  $B$ . If the number of turns per unit length in the coil is increased by a factor of 2, the magnitude of the magnetic field produced by the coil will be equal to which of the following?
- A.  $\frac{1}{2}B$
  - B.  $B$
  - C.  $2B$
  - D.  $4B$

Answer and Rationale

COMPETENCY 028

31. A  $4\ \Omega$  resistor and a  $12\ \Omega$  resistor are connected in parallel to a  $12\ \text{V}$  battery. The power dissipated by the circuit is equal to which of the following?
- A.  $48\ \text{W}$
  - B.  $18\ \text{W}$
  - C.  $9\ \text{W}$
  - D.  $4\ \text{W}$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 028

32. Which of the following uses mechanical energy to produce electrical energy?
- A. A transformer
  - B. A motor
  - C. A generator
  - D. A battery

Answer and Rationale

COMPETENCY 029

33. A railroad boxcar of mass  $M$  is moving at speed  $v$  along a straight horizontal track. It collides elastically with a second boxcar of the same mass  $M$  that is at rest. What are the magnitude of the total momentum and the total kinetic energy of the two boxcars immediately after the collision?

	<u>Momentum</u>	<u>Kinetic Energy</u>
A.	0	$\frac{1}{2} Mv^2$
B.	$Mv$	$\frac{1}{2} Mv^2$
C.	0	$Mv^2$
D.	$Mv$	$Mv^2$

Answer and Rationale

COMPETENCY 030

34. The second law of thermodynamics is specifically concerned with which of the following?
- A. The conservation of energy
  - B. The behavior of gases
  - C. The behavior of the entropy of a system
  - D. The molecular basis of temperature

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 030

35. When 100 J of heat energy is added to a system, the system does 40 J of work. What is the increase in the internal energy of the system?
- A. 40 J
  - B. 60 J
  - C. 100 J
  - D. 140 J

Answer and Rationale

COMPETENCY 031

36. A light ray passes from air ( $n = 1$ ) into glass ( $n = 1.5$ ). If the incident angle is  $30^\circ$ , what is the angle of refraction?
- A.  $30^\circ$
  - B.  $60^\circ$
  - C.  $\sin^{-1} \frac{1}{3}$
  - D.  $\sin^{-1} \frac{3}{4}$

Answer and Rationale

COMPETENCY 031

37. Which of the following is characteristic of light but not of sound?
- A. Dispersion
  - B. Constructive and destructive interference
  - C. Bending around an obstacle
  - D. Propagation in a vacuum

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

---

COMPETENCY 032

38. According to the Bohr model of the hydrogen atom, how does the energy of an electron depend on the principal quantum number  $n$ ?

- A. The energy of an electron is proportional to  $\frac{1}{n^2}$ .
- B. The energy of an electron is proportional to  $\frac{1}{n}$ .
- C. The energy of an electron is proportional to  $n$ .
- D. The energy of an electron is proportional to  $n^2$ .

Answer and Rationale

COMPETENCY 032

39. In the photoelectric effect, light is incident on the surface of a metal and causes photoelectrons to be emitted. For a given metal, which of the following is true about the energy of the emitted photoelectrons?

- A. The energy of the photoelectrons increases when the frequency of the incident light increases.
- B. The energy of the photoelectrons increases when the intensity of the incident light increases.
- C. The energy of the photoelectrons increases when the wavelength of the incident light increases.
- D. The energy of the photoelectrons is characteristic of the metal and independent of the incident light.

Answer and Rationale

COMPETENCY 033

40. Which of the following student responses is an example of correct conceptual understanding?

- A. The mass of an object is equal to its weight.
- B. The speed of light is greater than the speed of sound.
- C. Heavy objects always sink in water.
- D. The Moon and the Sun are the same size.

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

## Answer Key and Rationales

Question Number	Competency Number	Correct Answer	Rationales
1	001	A	<p><b>Option A is correct</b> because the distance light travels in one year is <math>(3.00 \times 10^8 \text{ m/sec})(3.16 \times 10^7 \text{ sec/yr}) = 9.48 \times 10^{15} \text{ m/yr}</math>. Therefore the distance from the star to our sun is <math>(1.08 \times 10^{17} \text{ m}) \div (9.48 \times 10^{15} \text{ m/yr}) \approx 11.39</math> light-years. <b>Options B and D are incorrect</b> because the calculation <math>(9.48 \times 10^{15} \text{ m/yr}) \div (1.08 \times 10^{17} \text{ m})</math> and <math>8.78 \times 10^{-2}</math> have units of <math>\frac{1}{\text{year}}</math>, instead of the correct unit <i>years</i>. <b>Option C is incorrect</b> because <math>(1.08 \times 10^{17} \text{ m}) \div (9.48 \times 10^{15} \text{ m/yr}) \approx 0.114 \times 10^2</math>, which is not equivalent to <math>1.14 \times 10^2</math>.</p> <p><a href="#">Back to Question</a></p>
2	002	B	<p><b>Option B is correct</b> because the argument of the product of two complex numbers is the sum of their arguments. The sum of the arguments of the two given complex numbers is slightly more than <math>\frac{\pi}{2}</math> radians. Only option B has the correct argument for the product. <b>Options A, C and D are incorrect</b> because the arguments of <math>z_1 z_2</math> shown are not the sum of the arguments of <math>z_1</math> and <math>z_2</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
3	003	C	<p><b>Option C is correct</b> because <math>p</math> divides both <math>n - 5</math> and <math>n + 6</math>, therefore <math>p</math> must also divide their difference, <math>(n + 6) - (n - 5) = 11</math>, and thus <math>p = 11</math>. Then <math>p</math> is also a factor of <math>n + 6 + 11k</math> and <math>n - 5 + 11k</math> for any integer <math>k</math>. Of the given options, only <math>n + 28 = n + 6 + 22 = n - 5 + 33</math> has this form. <b>Options A, B and D are incorrect</b> because these integers cannot be written in the form <math>n + 6 + 11k</math> or <math>n - 5 + 11k</math>, for some integer <math>k</math>.</p> <p><a href="#">Back to Question</a></p>
4	003	C	<p><b>Option C is correct</b> because the first term represents the value of the third deposit, the second term represents the value of the second deposit after two years and the third term represents the value of the first deposit after three years. The sum of these terms gives the total amount of money in the account at the end of the third year. <b>Options A and B are incorrect</b> because the compounding of the interest is not correct. <b>Option D is incorrect</b> because the terms <math>500(0.03)</math> give the value of the interest, not the principal and interest.</p> <p><a href="#">Back to Question</a></p>



Question Number	Competency Number	Correct Answer	Rationales
5	008	D	<p><b>Option D is correct</b> because the graph of <math>y = \log_2 x</math> has a vertical asymptote of the line <math>x = 0</math>, the <math>y</math>-axis. The function <math>y = 3 + \log_2(x - 4)</math> is a transformation of the graph of <math>y = \log_2 x</math>, with a vertical shift of 3 and a horizontal shift of 4. Thus, the vertical asymptote shifts horizontally from <math>x = 0</math> to <math>x = 4</math>. <b>Options A and B are incorrect</b> because the lines <math>y = 3</math> and <math>y = -4</math> are horizontal and cannot be vertical asymptotes. <b>Option C is incorrect</b> because the graph of <math>y = \log_2 x</math> is shifted horizontally by the transformation to the right, not the left.</p> <p><a href="#">Back to Question</a></p>
6	007	A	<p><b>Option A is correct</b> because the graph of the line <math>y = -3x - 2</math> intersects the graph of <math>y = x + 2</math> at the point <math>(-1, 1)</math>, and <math>-1 &lt; 2</math>. The graph of the line <math>y = -3x - 2</math> intersects the line <math>y = 10 - 2x</math> at the point <math>(-12, 34)</math>, but this point is not on the graph of the piecewise defined function because <math>-12</math> is not greater than 2. <b>Option B is incorrect</b> because the graph of the line <math>y = -3x - 2</math> does not intersect the graph of <math>y = x + 2</math> at the point <math>(-1, -5)</math>. <b>Option C is incorrect</b> because the graph of the line <math>y = -3x - 2</math> intersects the line <math>y = 10 - 2x</math> at the point <math>(-12, 34)</math>, but this point is not on the graph of the piecewise defined because <math>-12</math> is not greater than 2. <b>Option D is incorrect</b> because there is only one point of intersection between the two graphs.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
7	013	B	<p><b>Option B is correct</b> because the floor area consists of three rectangles, <math>8 \times 24</math>, <math>6 \times 18</math>, and <math>5 \times 6</math>, for a total area of 330 square feet. The tiles are each 1 square foot. Allowing for 10 percent breakage, Nick needs to purchase an additional <math>33 = 0.10(330)</math> tiles for a total of 363 tiles. <b>Option A is incorrect</b> because 330 square feet is the total area, and does not include extra tiles required in case of breakage. <b>Option C is incorrect</b> because 366 is the incorrect area derived from <math>14 \times 24</math> plus <math>5 \times 6</math>. <b>Option D is incorrect</b> because 402 is derived from the incorrect area of 366, plus 10 percent.</p> <p><a href="#">Back to Question</a></p>
8	014	D	<p><b>Option D is correct</b> because the first vector expressed in component form is <math>80i</math> and the second vector is <math>(60 \cos 40^\circ)i + (60 \sin 40^\circ)j</math>. The resultant is then <math>(80 + 60 \cos 40^\circ)i + (60 \sin 40^\circ)j</math>. <b>Options A and B are incorrect</b> because the component <math>80j</math> would result from walking due north rather than walking due east. <b>Option C is incorrect</b> because the second direction vector is <math>(60 \cos 40^\circ)i + (60 \sin 40^\circ)j</math> not <math>(60 \sin 40^\circ)i + (60 \sin 40^\circ)j</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
9	011	B	<p><b>Option B is correct</b> because the length of the curve <math>y = f(x)</math> between <math>x = a</math> and <math>x = b</math> is found by the integral <math>\int_a^b \sqrt{1 + (f'(x))^2} dx</math>. Thus, because <math>y' = 2x</math>, the length of the curve is <math>\int_0^2 \sqrt{1 + 4x^2} dx</math> over this interval. <b>Options A, C and D are incorrect</b> because they are not correct applications of the length of a curve formula.</p> <p><a href="#">Back to Question</a></p>
10	016	C	<p><b>Option C is correct</b> because <math>{}_{30}C_3</math> gives the number of ways to choose 3 students from the 30-person class, and <math>{}_{15}C_3</math> gives the number of ways of choosing 3 boys from among the 15 boys. Thus, the ratio <math>\frac{{}_{15}C_3}{{}_{30}C_3}</math> gives the probability that the teacher will choose 3 boys. <b>Options A, B and D are incorrect</b> because the numbers given do not correctly count the number of students to select.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
11	017	B	<p><b>Option B is correct</b> because the standard deviation of the sample means should be <math>\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{25}} = 0.04</math>. <b>Options A, C and D are incorrect</b> because the numbers given are not derived for the relevant formula.</p> <p><a href="#">Back to Question</a></p>
12	019	B	<p><b>Option B is correct</b> because the volume of a cylinder varies directly with the height when the radius is kept constant. The resulting graph will be a line with slope <math>(3^2)\pi = 9\pi</math> in the first-quadrant of the <math>xy</math>-plane through the origin.</p> <p><b>Option A is incorrect</b> because the line has a non-zero slope of <math>(3^2)\pi = 9\pi</math>.</p> <p><b>Options C and D are incorrect</b> because the volume of a cylinder varies directly with the height when the radius is kept constant and thus is not a polynomial of degree 2 or 3.</p> <p><a href="#">Back to Question</a></p>
13	018	C	<p><b>Option C is correct</b> because to establish deductively that the graph of a function is odd and has symmetry with respect to the origin, it suffices to establish that <math>f(-x) = -f(x)</math> for all values of <math>x</math> in the domain of the function. <b>Options A and B are incorrect</b> because the students may select only points where the conjecture is true and miss points that are counterexamples. <b>Option D is incorrect because</b> it is the criteria for an even function, not an odd function.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
14	021	A	<p><b>Option A is correct</b> because formative assessments are used at the beginning of, or during, instruction with the aim of refining instructional practices. Thus, they provide feedback to the teachers and students, and are not intended to be reported. <b>Options B, C and D are incorrect</b> because they do not describe formative assessments.</p> <p><a href="#">Back to Question</a></p>
15	011	C	<p><b>Option C is correct</b> because the ratio of <math>r</math> to <math>h</math> is equal to the ratio of the radius of the cone to the height of the cone, by the similarity of triangles. Because the radius of the cone is <math>\frac{10}{2} = 5</math>, and <math>\frac{r}{8} = \frac{5}{10}</math>, then <math>r = 4</math>. <b>Options A, B and D are incorrect</b> because a radius of these lengths would not satisfy the required ratio.</p> <p><a href="#">Back to Question</a></p>
16	001	C	<p><b>Option C is correct</b> because the growth rate in 1 hour is <math>\left(\frac{800 - 400}{400}\right) \times 100 = \frac{400}{400} \times 100 = 100\%</math>. <b>Option A is incorrect</b> because the growth is 400 bacteria per hour, not 400%. <b>Options B and D are incorrect</b> because these growth rates are not supported by the data.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
17	011	A	<p><b>Option A is correct</b> because the shaded area is found by subtracting the area under the graph of <math>g</math> from <math>a</math> to <math>b</math>, <math>\int_a^b g(x)dx</math>, from the area under the graph of <math>f</math> from <math>a</math> to <math>b</math>, <math>\int_a^b f(x)dx</math>. <b>Option B is incorrect</b> because the expression yields the negative of the area of the shaded region. <b>Option C is incorrect</b> because the expression represents the volume of the solid of revolution generated by revolving the shaded region about the <math>x</math>-axis. <b>Option D is incorrect</b> because the expression represents the volume of the solid of revolution generated by revolving the shaded region about the <math>y</math>-axis.</p> <p><a href="#">Back to Question</a></p>
18	010	D	<p><b>Option D is correct</b> because this is a direct application of the Fundamental Theorem of Calculus. <b>Options A, B and C are incorrect</b> because these are incorrect applications of the formulas for finding derivatives and calculating integrals.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
19	019	B	<p><b>Option B is correct</b> because if a principal of <math>P</math> dollars is invested at an annual interest rate <math>r</math>, compounded <math>n</math> times per year, and no further withdrawals from or deposits into the account are made, then the future value <math>A</math>, the account balance after <math>t</math> years, is given by the formula <math>A = P\left(1 + \frac{r}{n}\right)^{nt}</math>. Because Bank A compounds interest annually, the future value of \$10,000 invested in a savings account in Bank A after one year is <math>A = \\$10,000.00\left(1 + \frac{.05}{1}\right)^1 = \\$10,500.00</math>.</p> <p>Bank B compounds interest semiannually, so that the future value of \$10,000 invested at Bank B for one year is given by the formula</p> $B = \$10,000.00\left(1 + \frac{.05}{2}\right)^2 = \$10,506.25.$ <p>Thus, Samantha earns <math>\\$10,506.25 - \\$10,500.00 = \\$6.25</math> more interest in one year by investing in a savings account in Bank B. <b>Options A, C and D are incorrect</b> because they are derived from incorrect applications of the formula.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
20	015	C	<p><b>Option C is correct</b> because the chart shown is a histogram, and each bar displays a range of salaries. The chart shows that there are salaries of 61 students recorded. To calculate the possible average salary, find the weighted mean of the lowest possible salary and the highest possible salary, as shown in the chart. The weighted mean of the lowest possible salary is</p> $\frac{10(0) + 18(100) + 20(200) + 8(300) + 5(400)}{61} =$ $\frac{10,200}{61} = \$167.21$ <p>The weighted mean of the highest possible salary is</p> $\frac{10(99) + 18(199) + 20(299) + 8(399) + 5(499)}{61} =$ $\frac{16,239}{61} = \$266.21$ <p>Thus the average weekly salary is greater than \$167 and less than \$267.</p> <p><b>Option A is incorrect</b> because the range of the salaries cannot be determined from the information given; the highest and lowest salaries are not indicated on the chart. <b>Option B is incorrect</b> because the median salary is determined by ordering the salaries from least to greatest and finding the salary in the middle. There are 61 salaries, so the 31<sup>st</sup> salary is the median and will lie in the bar labeled \$200-\$299. <b>Option D is incorrect</b> because the mode is the salary that occurs most often in the list of salaries, but the individual values are not provided.</p> <p><a href="#">Back to Question</a></p>



Question Number	Competency Number	Correct Answer	Rationales
21	022	A, B, D	<p><b>Options A, B and D are correct</b> because the standard deviation is a measure of precision, repeatability or reproducibility. <b>Option C is incorrect</b> because it affects the true or actual value of a quantity (i.e., the accuracy).</p> <p><a href="#">Back to Question</a></p>
22	023	A, B, C	<p><b>Options A, B and C are correct</b> because theories are based on observations, experiments and empirical generalizations. <b>Option D is incorrect</b> because theories are not based on personal beliefs and speculations.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
23	025	D	<p><b>Option D is correct</b> because it correctly accounts for the vector nature of velocity. When the ball strikes the ground, it will have velocity components in both the horizontal and vertical directions. In the absence of air resistance, the external force in the horizontal direction is zero and the external force in the vertical direction is the downward force of gravity. Thus, the horizontal component of the ball's velocity, <math>v_x</math>, will remain constant throughout the motion at its initial value of <math>v_0</math>; that is, <math>v_x = v_0</math>. Also, the kinematic equations for constant acceleration, <math>y = \frac{1}{2}gt^2</math> and <math>v = gt</math>, can be used to find that the vertical component of the velocity, <math>v_y</math>, when the ball strikes the ground is equal to <math>v_y = \sqrt{2gh}</math>. Thus the speed of the ball when it strikes the ground is equal to the magnitude of the resultant of the two vectors, or <math>\sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gh}</math>.</p> <p><b>Option A is incorrect</b> because it is equal to the horizontal component of the velocity only. <b>Option B is incorrect</b> because it is equal to the vertical component of the velocity only. <b>Option C is incorrect</b> because it is equal to the sum of the horizontal and vertical components of the velocity, which is not equal to the magnitude of the resultant of the two vectors.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
24	025	A	<p><b>Option A is correct</b> because, according to Kepler’s third law, the period <math>T_1 = T</math> at radius <math>R_1 = R</math> is related to the period <math>T_2</math> at radius <math>R_2 = 2R</math> by the equation <math>\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3</math>, which gives <math>\left(\frac{T_2}{T}\right)^2 = \left(\frac{2R}{R}\right)^3 = 8</math>, or <math>T_2 = 2\sqrt{2}T</math>. <b>Options B, C and D are incorrect</b> because they do not agree with Kepler’s third law.</p> <p><a href="#">Back to Question</a></p>
25	025	C	<p><b>Option C is correct</b> because, according to Archimedes’ principle, the buoyant force acting on the block is equal to the weight of the volume of water displaced by the block, or <math>(1,000 \text{ kg/m}^3)(4 \times 10^{-4} \text{ m}^3)(10 \text{ m/s}^2) = 4 \text{ N}</math>. The tension in the wire is equal in magnitude to the apparent weight of the block which is equal to its actual weight minus the buoyant force, or <math>20 \text{ N} - 4 \text{ N} = 16 \text{ N}</math>. <b>Option A is incorrect</b> because it results from the assumption that the buoyant force is equal to the actual weight of the block. <b>Option B is incorrect</b> because it results from the assumption that the tension in the wire is equal to the buoyant force. <b>Option D is incorrect</b> because it results from the assumption that the tension in the wire is equal to the weight of the block.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
26	026	B	<p><b>Option B is correct</b> because, by Newton’s second law, the acceleration of the block is equal to the net force acting on the block divided by the mass of the block. The net force <math>F_{\text{net}}</math> on the block is equal to the 400 N force acting upward along the ramp minus the component of the block weight acting downward along the ramp, or <math>F_{\text{net}} = 400 \text{ N} - (40 \text{ kg})(10 \text{ m/s}^2)\sin 30^\circ = 400 \text{ N} - 200 \text{ N} = 200 \text{ N}</math>. Thus, by Newton’s second law, the acceleration <math>a</math> of the block is equal to <math>a = \frac{F_{\text{net}}}{m} = \frac{200 \text{ N}}{40 \text{ kg}} = 5 \text{ m/s}^2</math>. <b>Option A is incorrect</b> because a net force is acting on the block along the ramp, which means the block is accelerating. <b>Option C is incorrect</b> because it does not account for the component of the block’s weight along the ramp. <b>Option D is incorrect</b> because it results from using the wrong value for the net force acting on the block.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
27	026	A	<p><b>Option A is correct</b> because the net force acting on the moving block is equal to the applied force minus the force of kinetic friction, or <math>F_{net} = 50 \text{ N} - \mu_k Mg = 50 \text{ N} - (0.25)(10 \text{ kg})(10 \text{ m/s}^2) = 50 \text{ N} - 25 \text{ N} = 25 \text{ N}</math>. Thus, by Newton's second law, the acceleration of the block is equal to the net force divided by the mass of the block, or <math>\frac{F_{net}}{10 \text{ kg}} = \frac{25 \text{ N}}{10 \text{ kg}} = 2.5 \text{ m/s}^2</math>. <b>Option B is incorrect</b> because it does not account for the frictional force acting on the block. <b>Option C is incorrect</b> because it results from computing the net force as the sum, rather than the difference, of the applied force and the frictional force. <b>Option D is incorrect</b> because it is equal to the acceleration due to gravity, not to the acceleration of the block.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
28	027	A	<p><b>Option A is correct</b> because the period <math>T</math> of a simple pendulum is given by the equation <math>T = 2\pi\sqrt{\frac{L}{g}}</math> and, if the period on Jupiter is to be the same as the period on Earth, then it follows that <math>\frac{L}{g} = \frac{L_{\text{Jupiter}}}{g_{\text{Jupiter}}} = \frac{L_{\text{Jupiter}}}{2.5g}</math>, or <math>L_{\text{Jupiter}} = 2.5L</math>. <b>Option B is incorrect</b> because it results from the assumption that the acceleration due to gravity on Jupiter is equal to <math>\sqrt{2.5}g</math> instead of <math>2.5g</math>. <b>Option C is incorrect</b> because it results from the assumption that the length is unchanged, which would not give the same period on both planets. <b>Option D is incorrect</b> because it results from the assumption that the acceleration due to gravity on Jupiter is equal to <math>\frac{g}{2.5}</math> instead of <math>2.5g</math>.</p> <p><a href="#">Back to Question</a></p>
29	027	B	<p><b>Option B is correct</b> because, according to Kepler's second law, a planet sweeps out equal areas in equal times in its elliptical orbit about a star, which means that the speed of the planet must be greatest when it is closest to the star. <b>Options A and C are incorrect</b> because they are not in agreement with Kepler's second law. <b>Option D is incorrect</b> because the speed of the planet can be determined from known physical laws.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
30	027	C	<p><b>Option C is correct</b> because the magnitude <math>B</math> of the magnetic field produced by a coil of wire with <math>n</math> turns per unit length is given by <math>B = \mu nI</math>, where <math>\mu</math> is the magnetic permeability and <math>I</math> is the current in the coil. Thus, increasing <math>n</math> by a factor of 2 will increase the magnitude of the magnetic field by a factor of 2. <b>Options A, B and D are incorrect</b> because they result from assuming the wrong dependence of <math>B</math> on <math>n</math>.</p> <p><a href="#">Back to Question</a></p>
31	028	A	<p><b>Option A is correct</b> because two resistors, <math>R_1</math> and <math>R_2</math>, connected in parallel are equivalent to a single resistor given by <math>R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}</math>, and, according to Ohm's law, the power <math>P</math> dissipated by the circuit is equal to <math>P = IV = \frac{V^2}{R_{\text{eq}}}</math>. Thus,</p> $R_{\text{eq}} = \frac{(4\ \Omega)(12\ \Omega)}{4\ \Omega + 12\ \Omega} = \frac{48\ \Omega^2}{16\ \Omega} = 3\ \Omega$ <p>, which gives <math>P = \frac{(12\ \text{V})^2}{3\ \Omega} = 48\ \text{W}</math>. <b>Option B is incorrect</b> because it results from using the difference of the two resistors as the equivalent resistance to compute the power. <b>Option C is incorrect</b> because it results from using the sum of the two resistors as the equivalent resistance to compute the power. <b>Option D is incorrect</b> because it results from using <math>V</math> instead of <math>V^2</math> to compute the power.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
32	028	C	<p><b>Option C is correct</b> because a generator converts mechanical energy into electrical energy. <b>Option A is incorrect</b> because a transformer increases or decreases voltage through electromagnetic induction. <b>Option B is incorrect</b> because a motor converts electrical energy into mechanical energy. <b>Option D is incorrect</b> because a battery converts chemical energy into electrical energy.</p> <p><a href="#">Back to Question</a></p>
33	029	B	<p><b>Option B is correct</b> because momentum and kinetic energy are conserved in an elastic collision. The magnitude of the momentum before the collision is equal to the momentum <math>Mv</math> of the moving boxcar, which means that the magnitude of the total momentum of the two boxcars immediately after the collision must also be equal to <math>Mv</math>. The kinetic energy before the collision is equal to the kinetic energy <math>\frac{1}{2}Mv^2</math> of the moving boxcar, which means that the total kinetic energy of the two boxcars immediately after the collision must be equal to <math>\frac{1}{2}Mv^2</math>.</p> <p><b>Option A is incorrect</b> because the magnitude of the momentum immediately after the collision is not zero (the boxcars are not at rest). <b>Option C is incorrect</b> because immediately after the collision the magnitude of the momentum is not zero (the boxcars are not at rest) and the total kinetic energy is not equal to two times the kinetic energy before the collision. <b>Option D is incorrect</b> because the total kinetic energy immediately after the collision is not equal to two times the kinetic energy before the collision.</p> <p><a href="#">Back to Question</a></p>



Question Number	Competency Number	Correct Answer	Rationales
34	030	C	<p><b>Option C is correct</b> because the second law of thermodynamics is a statement about the entropy of a system. <b>Option A is incorrect</b> because the first law of thermodynamics, not the second law of thermodynamics, is a statement of the law of conservation of energy. <b>Option B is incorrect</b> because the behavior of gases is a consequence of the kinetic theory of gases, not of the second law of thermodynamics. <b>Option D is incorrect</b> because the molecular basis of temperature is a consequence of the kinetic theory of gases, not of the second law of thermodynamics.</p> <p><a href="#">Back to Question</a></p>
35	030	B	<p><b>Option B is correct</b> because, by the first law of thermodynamics, the internal energy increase is equal to the difference between the heat added to a system and the work done by the system, which in this case gives <math>100 \text{ J} - 40 \text{ J} = 60 \text{ J}</math>. <b>Option A is incorrect</b> because the increase in the internal energy of the system is not equal to the work done by the system. <b>Option C is incorrect</b> because the increase in the internal energy of the system is not equal to the heat added to the system. <b>Option D is incorrect</b> because the increase in the internal energy of the system is not equal to the sum of the heat added to the system and the work done by the system.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
36	031	C	<p><b>Option C is correct</b> because according to Snell's law, <math>n_{\text{air}} \sin 30^\circ = n_{\text{glass}} \sin \theta_{\text{glass}}</math>, where <math>\theta_{\text{glass}}</math> is the angle of refraction in glass.</p> <p>Thus, <math>\frac{1}{2} = 1.5 \sin \theta_{\text{glass}}</math>, or <math>\theta_{\text{glass}} = \sin^{-1} \frac{1}{3}</math>. <b>Option A is incorrect</b> because the refracted ray in the glass must be bent toward the normal, which means that the angle of refraction must be less than <math>30^\circ</math>. <b>Option B is incorrect</b> because the angle of refraction must be less than <math>30^\circ</math>. <b>Option D is incorrect</b> because it contains a mathematical mistake.</p> <p><a href="#">Back to Question</a></p>
37	031	D	<p><b>Option D is correct</b> because light does not require a medium in which to propagate, whereas sound waves are disturbances in a medium. <b>Options A, B and C are incorrect</b> because all waves, including light and sound waves, exhibit dispersion, constructive and destructive interference and bending around an obstacle.</p> <p><a href="#">Back to Question</a></p>
38	032	A	<p><b>Option A is correct</b> because, according to the Bohr model, the energy <math>E_n</math> of an electron in the <math>n</math>th energy level of the hydrogen atom is equal to <math>E_n = -\frac{13.6}{n^2}</math> eV. <b>Options B, C and D are incorrect</b> because they each have the wrong dependence on <math>n</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
39	032	A	<p><b>Option A is correct</b> because increasing the frequency of the incident light increases the energy of the emitted photoelectrons. <b>Option B is incorrect</b> because increasing the intensity of the incident light increases the number of emitted photoelectrons but has no effect on the energy of the photoelectrons. <b>Option C is incorrect</b> because decreasing, not increasing, the wavelength of the incoming light increases the energy of the emitted photoelectrons. <b>Option D is incorrect</b> because the energy of the emitted photoelectrons is not independent of the incident light.</p> <p><a href="#">Back to Question</a></p>
40	033	B	<p><b>Option B is correct</b> because the speed of light is much greater than the speed of sound. <b>Option A is incorrect</b> because mass is a measure of the amount of matter an object contains, while weight is the force of gravity on an object. <b>Option C is incorrect</b> because some heavy objects can float on water. <b>Option D is incorrect</b> because the Sun is much larger than the Moon.</p> <p><a href="#">Back to Question</a></p>



---

## Preparation Resources

The resources listed below may help you prepare for the TExES test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

### Mathematics

#### JOURNALS

*American Mathematical Monthly*, Mathematical Association of America.

*Journal for Research in Mathematics Education*, National Council of Teachers of Mathematics.

*Mathematics Magazine*, Mathematical Association of America.

*Mathematics Teacher*, National Council of Teachers of Mathematics.

#### OTHER RESOURCES

Bittenger, M. L., and Ellenbogen, D. (2008). *Elementary Algebra: Concepts and Applications*, Eighth Edition. Menlo Park, Calif.: Addison-Wesley.

Bock, D., Velleman, P., and De Veaux, R. (2009). *Stats: Modeling the World*, Third Edition. Boston, Mass.: Pearson Education, Inc.

Brahier, D. J. (2009). *Teaching Secondary and Middle School Mathematics*, Fourth Edition. Needham Heights, Mass.: Allyn & Bacon.

Brumbaugh, D. K., and Rock, D. (2013). *Teaching Secondary Mathematics*, Fourth Edition. Mahwah, N.J.: Lawrence Erlbaum Associates.

Burger, E., and Starbird, M. (2009). *The Heart of Mathematics: An Invitation to Effective Thinking*, Third Edition. Emeryville, Calif.: Key College Publishing.

COMAP (2011). *For All Practical Purposes: Mathematical Literacy in Today's World*, Ninth Edition. New York, N.Y.: W. H. Freeman & Company.

Connally, E., Hughes-Hallett, D., Gleason, A., et al. (2007). *Functions Modeling Change: A Preparation for Calculus*, Third Edition. Hoboken, N.J.: John Wiley & Sons, Inc.

Cooney, T., Brown, S., Dossey, J., Schrage, G., and Wittmann, E. (1999). *Mathematics, Pedagogy, and Secondary Teacher Education*. Portsmouth, N.H.: Heinemann.

Coxford, A., Usiskin, Z., and Hirschhorn, D. (2002). *The University of Chicago School of Mathematics Project: Geometry*. Glenview, Ill.: Scott, Foresman and Company.

Crouse, R. J., and Sloyer, C. W. (1987). *Mathematical Questions from the Classroom — Parts I and II*. Providence, R.I.: Janson Publications.

Danielson, C., and Marquez, E. (1998). *A Collection of Performance Tasks and Rubrics: High School Mathematics*. Larchmont, N.Y.: Eye on Education.

- 
- Demana, F., Waits, B. K., Clemens, S. R., and Foley, G. D. (1997). *Precalculus: A Graphing Approach*, Fourth Edition. Menlo Park, Calif.: Addison-Wesley.
- Foerster, P. A. (2005). *Calculus Concepts and Applications*, Second Edition. Berkeley, Calif.: Key Curriculum Press.
- Garfunkel, Godbold, and Pollak. (2010, 2011, 2013). *Mathematics: Modeling Our World. Books 1, 2 & 3*. New York, N.Y.: W. H. Freeman & Co.
- Greenburg, M. (2008). *Euclidean and Non-Euclidean Geometries: Development and History*, Fourth Edition. New York, N.Y.: W. H. Freeman and Company.
- Hughes-Hallett, D., Gleason, A., McCallum, W., et al. (2012). *Calculus: Single Variable*, Sixth Edition. Hoboken, N.J.: John Wiley & Sons, Inc.
- Hungerford, T. W. (2004). *Contemporary College Algebra and Trigonometry: A Graphing Approach*, Second Edition. Philadelphia, Pa.: Harcourt College Publishers.
- Johnson, K., Herr, T., Kysh, J. (2004). *Problem Solving Strategies: Crossing the River with Dogs*, Third Edition. Emeryville, Calif.: Key College Publishing.
- Kilpatrick, J., Swafford, J., and Finell, B. (Eds.) (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press.
- Kinsey, L., and Moore, T. (2008) *Symmetry, Shape, and Space: An Introduction to Mathematics Through Geometry*. Emeryville, Calif.: Key College Publishing.
- Larson, R., Hostetler, R., and Edwards, B. (2005). *Calculus of a Single Variable*, Eighth Edition. Boston, Mass.: Houghton Mifflin Harcourt.
- Lay, S. (2012). *Analysis: With an Introduction to Proof*, Fifth Edition. Upper Saddle River, N.J.: Pearson Education, Inc.
- Leitzel, James R. C. (1991). *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*. Washington, D.C.: Mathematical Association of America.
- National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- National Council of Teachers of Mathematics. (2009). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- Newmark, J. (1997). *Statistics and Probability in Modern Life*, Sixth Edition. Philadelphia, Pa.: Saunders College Publishing.
- Northey, S. (2005). *Handbook on Differentiated Instruction for Middle and High Schools*. Larchmont, N.Y.: Eye on Education.

- 
- Robbins, N. (2005). *Beginning Number Theory*, Second Edition. Sudbury, Mass.: Jones and Bartlett Publishers.
- Rosen, K. (2011). *Discrete Mathematics and Its Applications*, Seventh Edition. Boston, Mass.: McGraw-Hill Higher Education.
- Serra, M. (2008). *Discovering Geometry: An Investigative Approach*, Fourth Edition. Emeryville, Calif.: Key Curriculum Press.
- Shaughnessy, J., Chance, B., and Kranendonk, H. (2009). *Focus in High School Mathematics: Reasoning and Sense Making in Statistics and Probability*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- Strang, G. (2005). *Linear Algebra and its Applications*, Fourth Edition. Pacific Grove, Calif.: Brooks Cole Publishers.
- Texas Education Agency. (2009). Texas Essential Knowledge and Skills (TEKS).
- Triola, M. F. (2012). *Elementary Statistics*, Twelfth Edition. Boston, Mass.: Addison Wesley Longman, Inc.
- Usiskin, Z., Peressini, A., Marchisotto, E., and Stanley, D. (2003). *Mathematics for High School Teachers: An Advanced Perspective*. Upper Saddle River, N.J.: Pearson Education, Inc.
- Williams, G. (2004). *Applied College Algebra: A Graphing Approach*, Second Edition. Philadelphia, Pa.: Harcourt College Publishers.
- Wolf, R. (1998). *Proof, Logic, and Conjecture: The Mathematician's Toolbox*. New York, N.Y.: W. H. Freeman and Company.
- Wright, D. (1999). *Introduction to Linear Algebra*. Boston, Mass.: WCB McGraw-Hill.

### **ONLINE RESOURCES**

- Mathematical Association of America — [www.maa.org](http://www.maa.org)
- Math Forum at Drexel — [www.mathforum.org](http://www.mathforum.org)
- National Council of Teachers of Mathematics — [www.nctm.org](http://www.nctm.org)
- National Research Council — <http://sites.nationalacademies.org/NRC>
- Wolfram MathWorld — [www.mathworld.wolfram.com](http://www.mathworld.wolfram.com)

---

## Physics

### JOURNALS

*American Scientist*, Sigma XI, the Scientific Research Society.

*Texas Science Teacher*, Science Teachers Association of Texas.

*The Physics Teacher*, American Association of Physics Teachers.

*The Science Teacher*, National Science Teachers Association.

### OTHER RESOURCES

Arons, A. B. (1996). *Teaching Introductory Physics*. New York, N.Y.: Wiley.

Center for Science, Mathematics, and Engineering Education: National Research Council. (2000). *Inquiry and the National Science Education Standards: A Guide for Teaching and Learning*. Washington, D.C.: National Academy Press.

Chiappetta, E. L., and Koballa, T. R. (2010). *Science Instruction in the Middle and Secondary Schools*, Seventh Edition. Upper Saddle River, N.J.: Pearson.

Crotts, D. (1995). *Critical Thinking Skills: Science*. Torrance, Calif.: Frank Schaffer Publications, Inc.

Cunningham, J., and Herr, N. (1994). *Hands-on Physics Activities with Real-Life Applications: Easy-to-Use Labs and Demonstrations for Grades 8–12*. San Francisco, Calif.: Jossey-Bass.

Ebenezer, J. V., and Haggerty, S. M. (1999). *Becoming a Secondary School Science Teacher*, First Edition. Upper Saddle River, N.J.: Prentice-Hall.

Haber-Schaim, U., et al. (1991). *PSSC Physics*, Seventh Edition. Dubuque, Iowa: Kendall/Hunt.

Halliday, D., Resnick, R., and Walker, J. (2013). *Fundamentals of Physics*, Tenth Edition. Hoboken, N.J.: Wiley.

Hewitt, P. G., Suchocki, J. A., and Hewitt, L. A. (2011). *Conceptual Physical Science*, Fourth Edition. Boston, Mass.: Addison-Wesley Publishing Company, Inc.

Joyce, B. R., and Weil, M. (2008). *Models of Teachings*, Eighth Edition. Upper Saddle River, N.J.: Pearson.

National Research Council. (1998). *National Science Education Standards*, Fifth Printing. Washington, D.C.: National Academy Press.

Ostlund, K. L. (1992). *Science Process Skills: Assessing Hands-On Student Performance*. Menlo Park, Calif.: Addison-Wesley Publishing Company, Inc.

Project 2061 (American Association for the Advancement of Science). (1994). *Benchmarks for Science Literacy*. New York, N.Y.: Oxford University Press.

Rakow, S. J. (Ed.). (2000). *NSTA Pathways to the Science Standards: Guidelines for Moving the Vision into Practice*, Middle School Edition. Arlington, Va.: National Science Teachers Association.

Ramig, J., Bailer, J., and Ramsey, J. (1995). *Teaching Science Process Skills*. Torrance, Calif.: Frank Schaffer Publications, Inc.



---

Rezba, R. J., et al. (2010). *Learning and Assessing Science Process Skills*, Fifth Edition. Dubuque, Iowa: Kendall/Hunt Publishing Company.

Santa, C. M., and Alverman, D. E. (Eds.). (1991). *Science Learning: Processes and Applications*. Newark, Del.: International Reading Association, Inc.

Serway, R. A., and Faughn, J. S. (2011). *College Physics*, Ninth Edition. Boston, Mass.: Brooks Cole.

Texas Education Agency. (2010). *Texas Essential Knowledge and Skills (TEKS)*.

Texas Education Agency. (2010). *Texas Safety Standards: Kindergarten through Grade 12*, Fourth Edition.

Tipler, P. A., and Llewellyn, R. A. (2012). *Modern Physics*, Sixth Edition. New York, N.Y.: W. H. Freeman.

Trowbridge, L. W., Bybee, R. W., and Powell, J. C. (2008). *Teaching Secondary School Science: Strategies for Developing Scientific Literacy*, Ninth Edition. Upper Saddle River, N.J.: Pearson.

### **ONLINE RESOURCES**

American Association for the Advancement of Science — [www.aaas.org](http://www.aaas.org)

American Association of Physics Teachers — [www.aapt.org](http://www.aapt.org)

American Physical Society — [www.aps.org](http://www.aps.org)

National Science Teachers Association — [www.nsta.org](http://www.nsta.org)